Pressure Profile in Incompressible Quasi-Isothermal Proxy Pipe Flow @model

Motivation

One of the key challenges in Pipe Flow Dynamics is to predict the pressure distribution along the pipe during the stationary fluid transport.

In many practical cases the stationary pressure distribution can be approximated by Isothermal or Quasi-isothermal homogenous fluid flow model.

Pipeline Flow Pressure Model is addressing this problem with account of the varying pipeline trajectory, gravity effects and fluid friction with pipeline walls.

Outputs

$p(l)$	Pressure distribution along the pipe
$q(l)$	Flowrate distribution along the pipe
$u(l)$	Flow velocity distribution along the pipe

Inputs

T_0	Intake temperature	$T(l)$	Along-pipe temperature profile
p_0	Intake pressure	$\rho(T,p)$	Fluid density
q_0	Intake flowrate	$\mu(T,p)$	Fluid viscosity
$z(l)$	Pipeline trajectory TVDss	A	Pipe cross-section area
$\theta(l)$	Pipeline trajectory inclination, $\cos\theta(l) = \frac{dz}{dl}$	ϵ	Inner pipe wall roughness

Assumptions

Steady-State flow	Isothermal or Quasi-isothermal flow
$\frac{\partial p}{\partial t} = 0$	$\frac{\partial T}{\partial t} = 0 \rightarrow T(t,l) = T(l)$
Homogenous flow	Constant cross-section pipe area A along hole
$\frac{\partial p}{\partial \tau_x} = \frac{\partial p}{\partial \tau_y} = 0 \rightarrow p(t, \tau_x, \tau_y, l) = p(l)$	$A(l) = A = \mathrm{const}$
Incompressible fluid	

$$\rho(T,p) = \rho_0 = \text{const} \quad \mu(T,\rho) = \mu_0 = \text{const}$$

Equations

Pressure profile	Pressure gradient profile
(1) $p(l) = p_0 + g \int_0^l \rho(l) \cdot \cos\theta(l)\, dl - \frac{j_m^2}{2d} \int_0^l \frac{f(l)\, dl}{\rho(l)}$	(2) $\frac{dp}{dl} = \rho(l)\, g \cos\theta(l) - \frac{j_m^2}{2d} \frac{f(l)}{\rho(l)}$

where

j_m	Intake mass flux
$\dot m$	mass flowrate
$u_0 = u(l=0)$	Intake Fluid velocity
$\Delta z(l) = z(l) - z(0)$	elevation drop along pipe trajectory
$f_0 = f(\text{Re}_0, \epsilon)$	Darcy friction factor at intake point
$\text{Re}_0 = \frac{u(l)\cdot d}{\nu(l)} = \frac{4\rho_0 q_0}{\pi d} \frac{1}{\mu_0}$	Reynolds number at intake point
$d = \sqrt{\frac{4A}{\pi}}$	characteristic linear dimension of the pipe (or exactly a pipe diameter in case of a circular pipe)

Incompressible fluid $\rho(T,p) = \rho_0 = \text{const}$ means that compressibility vanishes $c(p) = 0$ and fluid velocity is going to be constant along the pipeline trajectory $u(l) = u_0 = \frac{q_0}{A} = \text{const}$.

For the constant viscosity $\mu(T,p) = \mu_0 = \text{const}$ along the pipeline trajectory the Reynolds number $\text{Re}(l) = \frac{j_m^2\, d}{\mu_0} = \text{const}$ and Darcy friction factor $f(l) = f(\text{Re}, \epsilon) = f_0 = \text{const}$ are going to be constant along the pipeline trajectory.

Equation (3) becomes:

(3) $$\frac{dp}{dl} = \rho_0\, g \frac{dz}{dl} - \frac{j_m^2 f_0}{2\,\rho_0\, d}$$

which leads to (1) after substituting $\cos\theta(l) = \frac{dz(l)}{dl}$ and can be explicitly integrated leading to

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The first term in the right side of (1) defines the hydrostatic column of static fluid while the last term defines the friction losses under fluid movement:

In most practical applications in water producing or water injecting wells, water can be considered as incompressible and friction factor can be assumed constant $f(l) = f_0 = \mathrm{const}$ along-hole (see Darcy friction factor in water producing /injecting wells).

See also

Physics / Fluid Dynamics / Pipe Flow Dynamics / Pipe Flow Simulation / Pressure Profile in Homogeneous Quasi-Isothermal Steady-State Pipe Flow @model

[Pressure Profile in Incompressible Isothermal Proxy Pipe Flow @model]

[Darcy friction factor] [Darcy friction factor @model]

[Homogenous Pipe Flow Temperature Profile @model]

References