

# Derivation of linear single-phase pressure diffusion @model

We start with the general form of ([Single-phase pressure diffusion @model:1](#)):

(1) $\phi \cdot c_t \cdot \partial_t p + \nabla \mathbf{u} + \mathbf{c} \cdot (\mathbf{u} \nabla p) = \sum_k q_k(t) \cdot \delta(\mathbf{r} - \mathbf{r}_k)$	(2) $\mathbf{u} = -M \cdot (\nabla p - \rho \mathbf{g})$	(3) $\int_{\Gamma} \mathbf{u} d\Sigma = q_{\Gamma}(t)$
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where

$p(t, \mathbf{r})$	reservoir pressure	$t$	time
$\rho(\mathbf{r})$	fluid density	$\mathbf{r}$	position vector
$\phi(\mathbf{r})$	effective porosity	$\mathbf{r}_k$	position vector of the $k$ -th source
$c_t(\mathbf{r})$	total compressibility	$\delta(\mathbf{r})$	Dirac delta function
$M(\mathbf{r})$	reservoir fluid mobility $M(\mathbf{r}) = \frac{k(\mathbf{r})}{\mu}$	$\nabla$	gradient operator
$k(\mathbf{r})$	formation permeability to a given fluid	$\mathbf{g}$	gravity vector
$\mu$	dynamic viscosity of a given fluid	$\mathbf{u}$	fluid velocity under Darcy flow
$q_k(t)$	sandface flowrates of the $k$ -th source	$\Gamma$	reservoir boundary
$q_{\Gamma}(t)$	flow through the reservoir boundary $\Gamma$ , which is aquifer or gas cap		

Let's neglect the non-linear term  $\mathbf{c} \cdot (\mathbf{u} \nabla p)$  for low compressibility fluid  $c \sim 0$  which is equivalent to assumption of nearly constant fluid density:  $\rho(p) = \rho = \text{const.}$

Together with constant pore compressibility  $c_{\phi} = \text{const}$  this will lead to constant total compressibility  $c_t = c_{\phi} + c \approx \text{const.}$

Assuming that permeability and fluid viscosity do not depend on pressure  $k(p) = k = \text{const}$  and  $\mu(p) = \mu = \text{const}$  one arrives to the differential equation with constant coefficients:

(4) $\phi \cdot c_t \cdot \partial_t p + \nabla \mathbf{u} = \sum_k q_k(t) \cdot \delta(\mathbf{r} - \mathbf{r}_k)$	(5) $\mathbf{u} = -M \cdot (\nabla p - \rho \mathbf{g})$	(6) $\int_{\Gamma} \mathbf{u} d\Sigma = q_{\Gamma}(t)$	
or			
(7) $\phi \cdot c_t \cdot \partial_t p + \nabla \mathbf{u} = 0$	(8) $\mathbf{u} = -M \cdot (\nabla p - \rho \mathbf{g})$	(9) $\int_{\Sigma_k} \mathbf{u} d\Sigma = q_k(t)$	(10) $\int_{\Gamma} \mathbf{u} d\Sigma = q_{\Gamma}(t)$

## See also

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[Physics / Mechanics / Continuum mechanics / Fluid Mechanics / Fluid Dynamics / Pressure Diffusion / Pressure Diffusion @model](#) / [Single-phase pressure diffusion @model](#)

