Pressure Profile in G-Proxy Pipe Flow @model

Motivation

Numerical quadrature solution of Pressure Profile in Homogeneous Steady-State Pipe Flow @model

Outputs

$p(l)$	Pressure distribution along the pipe
$q(l)$	Flowrate distribution along the pipe
$u(l)$	Flow velocity distribution along the pipe

Inputs

T_0	Intake temperature	$T(l)$	Along-pipe temperature profile
p_0	Intake pressure	$\rho(T,p)$	Fluid density
q_0	Intake flowrate	$\mu(T,p)$	Fluid viscosity
$z(l)$	Pipeline trajectory TVDss	A	Pipe cross-section area
$\theta(l)$	Pipeline trajectory inclination, $\cos\theta(l) = \frac{dz}{dl}$	ϵ	Inner pipe wall roughness

Assumptions

Steady-State flow	Quasi-isothermal flow
$\frac{\partial p}{\partial t} = 0$	$\frac{\partial T}{\partial t} = 0 \rightarrow T(t,l) = T(l)$
Homogenous flow	Constant cross-section pipe area A along hole
$\frac{\partial p}{\partial \tau_x} = \frac{\partial p}{\partial \tau_y} = 0 \rightarrow p(t, \tau_x, \tau_y, l) = p(l)$	$A(l) = A = \text{const}$
Constant inclination	
$\theta(l) = \theta = \text{const} \rightarrow \cos\theta = \frac{dz}{dl} = \text{const}$	

Equations

Pressure profile along the pipe
(1) $L = L(p) = \int_{p_0}^{p} \frac{\rho(p) - j_m^2\, c(p)}{G\, \rho^2(p) - F(\rho(p))}\, dp$

where

$j_m = \frac{\dot m}{A} = \mathrm{const}$	mass flux
$\dot m = \frac{dm}{dt} = \mathrm{const}$	mass flowrate
$q_0 = \frac{dV_0}{dt} = \frac{\dot m}{\rho_0}$	Intake volumetric flowrate
$\rho_0 = \rho(T_0, p_0)$	Intake fluid density
$\Delta z(l) = z(l) - z(0)$	elevation drop along pipe trajectory
$f(T, \rho) = f(\mathrm{Re}(T, \rho), \epsilon)$	Darcy friction factor
$\mathrm{Re}(T, \rho) = \frac{j_m \cdot d}{\mu(T, \rho)}$	Reynolds number in Pipe Flow
$\mu(T, \rho)$	dynamic viscosity as function of fluid temperature T and density ρ
$c(T, p) = \frac{1}{\rho} \left(\frac{\partial \rho}{\partial p} \right)_T$	fluid compressibility
$d = \sqrt{\frac{4A}{\pi}} = \mathrm{const}$	characteristic linear dimension of the pipe (or exactly a pipe diameter in case of a circular pipe)
$G = g\, \cos\theta = \mathrm{const}$	gravity acceleration along pipe
$F(T, \rho) = j_m^2 \cdot f(T, \rho)/(2d)$	

See Derivation of Pressure Profile in G-Proxy Pipe Flow @model

Alternative forms

Density form
(2) $L = L(\rho) = \int_{\rho_0}^{\rho} \frac{1/c(\rho) - j_m^2/\rho}{G\, \rho^2 - F(\rho)}\, d\rho$
Pressure-Density form

(3) $$L = \int_{p_0}^{p} \frac{\rho \, dp}{G \, \rho^2 - F(\rho)} - j_m^2 \cdot \int_{\rho_0}^{\rho} \frac{1}{\rho} \frac{d\rho}{G \, \rho^2 - F(\rho)}$$

This form is useful for derivation of Pressure Profile in GF-Proxy Pipe Flow @model

Approximations

Pressure Profile in GF-Proxy Pipe Flow @model

See also

Physics / Fluid Dynamics / Pipe Flow Dynamics / Pipe Flow Simulation / Pressure Profile in Homogeneous Quasi-Isothermal Steady-State Pipe Flow @model

[Pressure Profile in GF-Proxy Pipe Flow @model]