

Pressure Line Source Solution (LSS) @model

Motivation

Reservoir pressure dynamics away from [wellbore](#) and boundaries is sensitive to the two specific [complex reservoir properties](#): [transmissibility](#) σ and [pressure diffusivity](#) χ .

In case the [reservoir flow](#) has been created by a [well](#) (vertical or horizontal) it will trend to form a [radial flow](#) away from boundaries and [well](#) itself.

In this case a [pressure drop](#) and [well flowrate](#) can be roughly related to each other by means of a simple analytical homogeneous reservoir flow model with [wellbore](#) and boundary effects neglected.

Since the [well radius](#) is neglected the [well](#) is modeled as a vertical 0-thickness line, sourcing the fluid from a reservoir, giving a model a specific name [Line Source Solution](#).

Inputs & Outputs

Inputs		Outputs	
q_i	total sandface rate	$p(t, r)$	reservoir pressure
p_i	initial formation pressure		
σ	transmissibility		
χ	pressure diffusivity		

$\sigma = \frac{k h}{\mu}$	transmissibility		μ	dynamic fluid viscosity
$\chi = \frac{k}{\mu} \frac{1}{\phi c_t}$	pressure diffusivity		t	time
k	absolute permeability		r	radial direction
ϕ	porosity		c_t	total compressibility

Physical Model

Radial fluid flow	Homogenous reservoir	Infinite boundary	Zero wellbore radius	Slightly compressible fluid flow	Constant rate production
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$p(t, r)$	$M(r, p) = M = \text{const}$ $\phi(r, p) = \phi = \text{const}$ $h(r) = h = \text{const}$ $c_r(r) = c_r = \text{const}$	$r \rightarrow \infty$	$r_w = 0$	$c_t(p) = c_r + c = \text{const}$	$q_t = \text{const}$
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Mathematical Model

Motion equation		Initial condition	Boundary conditions	
(1)	$\frac{\partial p}{\partial t} = \chi \left[\frac{\partial^2 p}{\partial r^2} + \frac{1}{r} \frac{\partial p}{\partial r} \right]$	(2) $p(t = 0, r) = p_i$	(3) $p(t, r = \infty) = p_i$	(4) $\left[r \frac{\partial p}{\partial r} \right]_{r=0} = \frac{q_t}{2\pi\sigma}$

Computational Model

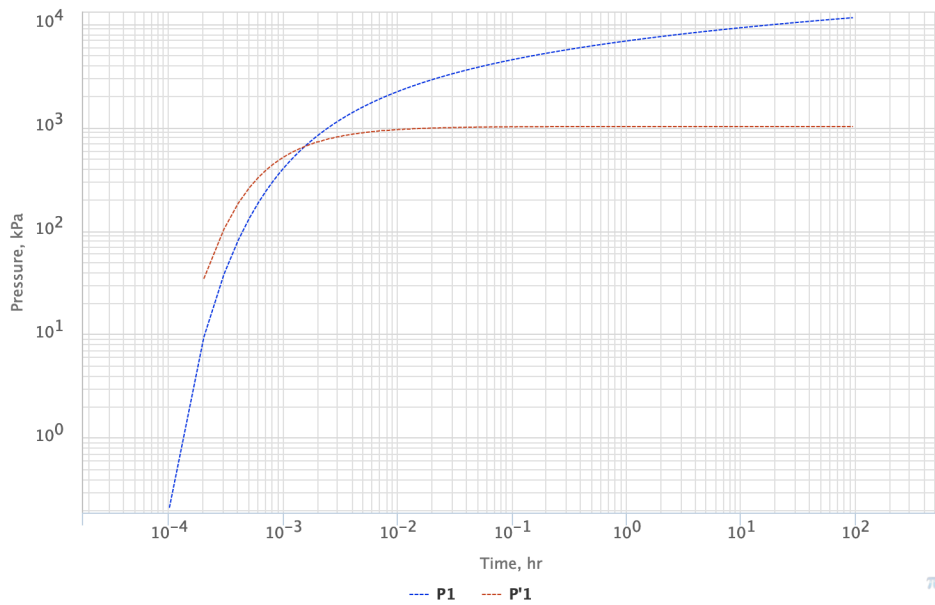
(5) $p(t, r) = p_i + \frac{q_t}{4\pi\sigma} \text{Ei} \left(-\frac{r^2}{4\chi t} \right)$	$\text{Ei}(\xi)$ – exponential integral
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Approximations

Infinite Acting Radial Flow(IARF)
$t \gg \frac{r^2}{4\chi}$
(6) $p(t, r) \sim p_i + \frac{q_t}{4\pi\sigma} \left[\gamma + \ln \left(\frac{r^2}{4\chi t} \right) \right] = p_i - \frac{q_t}{4\pi\sigma} \ln \left(\frac{2.24585 \chi t}{r^2} \right)$

Diagnostic Plots

Pressure Drop	
(7)	$\delta p = p_i - p_{wf}(t) = \frac{q_t}{4\pi\sigma} \text{Ei} \left(-\frac{r^2}{4\chi t} \right)$



$$(9) \quad \delta p \sim \ln t + \text{const}, \quad t \gg \frac{r^2}{4\chi}$$

Fig. 1. PTA Diagnostic Plot for LSS pressure response for the 0.1 md reservoir in a close line source vicinity (0.1 m), which is about a typical wellbore size.

One can easily see that with wellbore effects neglected even for a very low permeability reservoir the IARF regime is getting formed very early at 0.01 hr (36 s).

See also

[Physics / Fluid Dynamics / Radial fluid flow](#)

[[Radial Flow Pressure @model](#)] [[1DR pressure diffusion of low-compressibility fluid](#)] [[Exponential Integral](#)]