How to Calculate the Free Convection Coefficient for Vertical or Horizontal Isothermal Planes

The free convection coefficient can be described in terms of dimensionless groups. Familiarize yourself with the dimensionless groups described in Table 1 before continuing on with the procedures listed below for calculating the free convection coefficient.

Table 1: Dimensionless groups of importance for heat transfer and fluid flow

Group	Definition	Interpretation
Biot number (Bi)	$\frac{\bar{h}L}{k_{solid}}$	Ratio of internal thermal resistance of a solid body to its surface thermal resistance
Grashof number (Gr_L)	$\frac{g\beta(T_w - T_\infty)L^3}{\nu^2}$	Ratio of buoyancy to viscous forces
Nusselt number (Nu_L)	$\frac{\bar{h}L}{k_{fluid}}$	Dimensionless heat transfer coefficient; ratio of convection heat transfer to conduction in a fluid layer of thickness *L*
Prandtl number (Pr)	$\frac{c_p\mu}{k} = \frac{\nu}{\alpha}$	Ratio of molecular momentum diffusivity to thermal diffusivity

Procedure for Calculating the Free Convection Coefficient of Vertical Isothermal Planes

The film temperature is given as

$$T_f = \frac{T_\infty + T_w}{2}$$

where T_∞ represents the temperature of the environment and T_w represents the wall temperature. The Grashof number is found using

$$Gr = \frac{g\beta(T_w - T_\infty)L^3}{\nu^2}$$

where g is the gravitational constant, L is the length of the vertical surface, ν is the kinematic viscosity of the convective fluid evaluated at T_f and β represents the temperature coefficient of thermal conductivity. The Rayleigh is given by

$$Ra = GrPr$$

where Pr represents the Prandtl number of the convective fluid at T_f. The average Nusselt number given by Churchill and Chu (1975) is

$$\overline{Nu} = 0.68 + \frac{0.670\ Ra^{1/4}}{[1 + (0.492/Pr)^{9/16}]^{4/9}} \qquad \text{for } Ra_L < 10^9$$

$$\overline{Nu}^{1/2} = 0.825 + \frac{0.387\ Ra^{1/6}}{[1 + (0.492/Pr)^{9/16}]^{8/27}} \qquad \text{for } 10^{-1} < Ra_L < 10^{12}$$

The average free-convection heat transfer coefficient is given by

$$\bar{h} = \frac{\overline{Nu}k}{L}$$

where k is the thermal conductivity of the convective fluid at T_f.

Procedure for Calculating the Free Convection Coefficient of Horizontal Isothermal Planes

The film temperature is given as

$$T_f = \frac{T_\infty + T_w}{2}$$

where T_∞ represents the temperature of the environment and T_w represents the wall temperature. The Grashof number is found using

$$Gr = \frac{g\beta(T_w - T_\infty)L^3}{\nu^2}$$

where

$$L = \frac{A}{P}$$

and where g is the gravitational constant, A is the area of the horizontal surface, P is the perimeter of the horizontal surface, ν is the kinematic viscosity of the convective fluid evaluated at T_f and β represents the temperature coefficient of thermal conductivity (1/ T_f where T_f is in K). The average Nusselt number follows the functional form

$$\overline{Nu} = C(GrPr)^m$$

where Pr is the Prandtl number of the convective fluid at T_f and C and m are constants to be read from Table 2.

Table 2: Constants used to calculate average Nusselt number for isothermal surfaces

Geometry	$GrPr$	C	m	Reference(s)
Upper surface of heated plate or lower surface of cooled plate	$< 2 \times 10^8$	0.13	$\frac{1}{3}$	Fujii and Imura (1972)
Upper surface of heated plate or lower surface of cooled plate	$2 \times 10^8 – 10^{11}$	0.16	$\frac{1}{3}$	Fujii and Imura (1972)
Lower surface of heated plate or upper surface of cooled plate	$10^6 – 10^{11}$	0.58	$\frac{1}{4}$	Fujii and Imura (1972)

The average free-convection heat transfer coefficient is given by

$$\bar{h} = \frac{\overline{Nu}k}{L}$$

where k is the thermal conductivity of the convective fluid at T_f.

Example: Calculating the Free Convection Coefficient of Vertical Isothermal Planes

Consider the square shown in Fig. 1. The top face, top-right corner and right face are exposed to air at 22.22°C. Calculate the convection coefficient associated with node 4.

The nodal size and spacing between nodes is 0.1 m. .

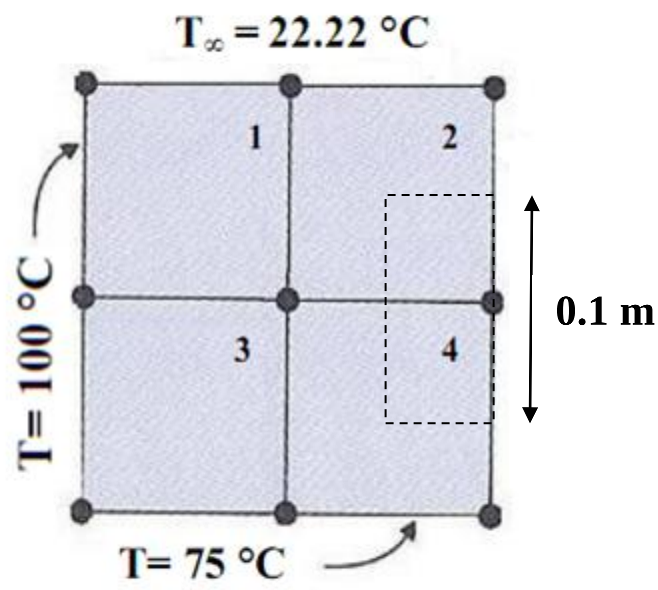

Figure 1: Example 2

Example 1 Solution:

Determine the film temperature at node 4:

$$T_f = \frac{22.22\ °C + T_4}{2}$$

An initial guess of the temperature at node 4 is made because T_4 unknown. The temperature at T_4 can be updated once the convection coefficient is evaluated through iteration techniques. For now T_4 = 60°C.

$$T_f = \frac{22.22\ °C + 60°C}{2} = 41.11°C = 314.26\ K$$

The properties of interest were evaluate for air at $41.11°C$

$$\beta = \frac{1}{314.26\ K} = 0.00318 \qquad \nu = 1.712 \times 10^{-5}\, m^2/s$$

$$k = 0.0274\ W/m\cdot°C \qquad \mathrm{Pr} = 0.705$$

$$Gr = \frac{(9.81\ m/s^2)(0.00318\ K^{-1})(333.15\ K - 295.37\ K)(0.1\ m)^3}{(1.712 \times 10^{-5}\, m^2/s)^2} = 4023781$$

$$Ra = (4023781)(0.705) = 2836766$$

$$\overline{Nu} = 0.68 + \frac{0.670\ (2836766)^{1/4}}{[1 + (0.492/(0.705))^{9/16}]^{4/9}} = 21.767$$

$$\bar{h} = \frac{(21.767)(0.0274\ W/m\cdot°C)}{0.1\ m} = 5.96\ W/m^2\cdot°C$$

References

Churchill, S. W., and H. H. S. Chu. "Correlating Equations for Laminar and Turbulent Free Convection from a Vertical Plate," *Int. J. Heat Mass Transfer*, vol. 18, p. 1323, 1975.

Fujii, T., and H. Imura. "Natural Convection Heat Transfer from a Plate with Arbitrary Inclination," *Int. J. Heat and Mass Transfer*, vol. 15, p. 755, 1972.

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