Logarithmic Derivative @model

The calculation algorithm of Logarithmic Derivative uses the weighted slopes $\Delta P / \Delta X$ of the pressure change versus change in time:

(1)
$$\left(\frac{dP}{dX}\right)_j = \frac{\frac{\Delta P_1}{\Delta X_1}\Delta X_2 + \frac{\Delta P_2}{\Delta X_2}\Delta X_1}{\Delta X_1 + \Delta X_2},$$

where $\Delta P_1 = P_j - P_{j-1}$, $\Delta P_2 = P_{j+1} - P_j$ are the functions of the pressure change with respect to the point of interest and $\Delta X_1 = X_j - X_{j-1}$, $\Delta X_2 = X_{j+1} - X_j$ are the functions of the change in time.

The straightforward application of the (1) formula results in the noisy data.

The most popular practical solution is to apply Sparsing and Smoothing.

Sparsing

Given the approved point $X_{j \in (1..N)}$, the next approved point $X_{j+k}|_{k=1,2,3,...}$ is going to be when $X_{j+k} - X_j > \frac{1}{10^{n+1}}$ holds true, where:

N	total number of source data points
n	number of points per logarithmic cycle

Suppose N is the number of data points, n is the number of points per logarithmic cycle, and X_j is the j-th data point, $\mathrm{j} = 1, 2, \ldots, \mathrm{N}$. If inequality $X_{j+k} - X_j < \frac{1}{10^{n+1}}$, where $k = 1, 2, 3, \ldots,$ is valid, then points X_{j+k} are removed.

Smoothing

In order to smooth the derivative, the so-called 'L-spacing' method is applied. Suppose the smoothing parameter L is given. For every $j = 2 \ldots N - 1$ there exist points $m = j - k_1$ and $n = j + k_2$ such that $X_n - X_j > L$ and $X_j - X_m > L$. Thus, the logarithmic derivative formula changes to:

(2)
$$\left(\frac{d\hat{P}}{dX}\right)_j = \frac{\frac{\Delta \hat{P}_1}{\Delta \hat{X}_1}\Delta \hat{X}_2 + \frac{\Delta \hat{P}_2}{\Delta \hat{X}_2}\Delta \hat{X}_1}{\Delta \hat{X}_1 + \Delta \hat{X}_2},$$

where $\Delta \hat{P}_1 = P_j - P_m$, $\Delta \hat{P}_2 = P_n - P_j$, $\Delta \hat{X}_1 = X_j - X_m$, $\Delta \hat{X}_2 = X_n - X_j$. In practice, it is recommended to choose L between 0 and 0.5.

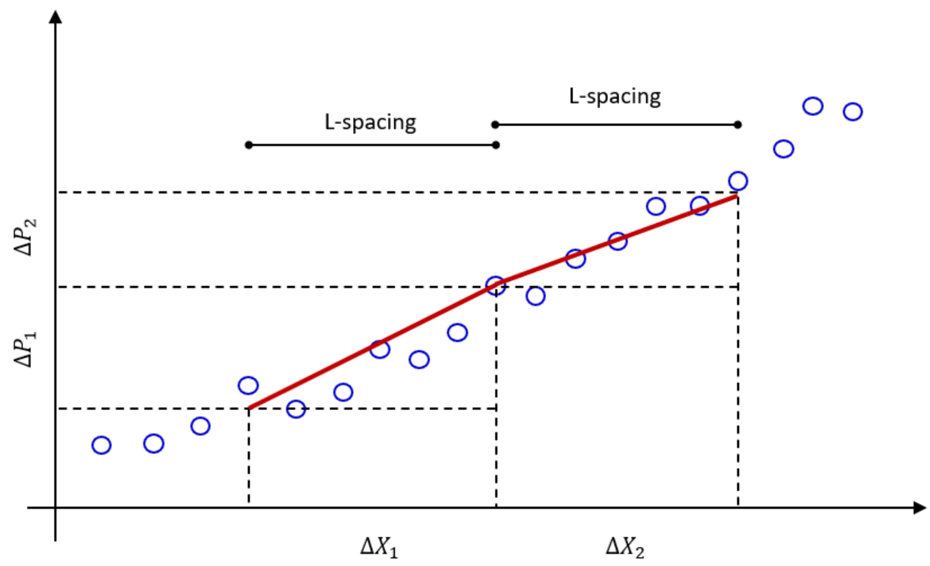

Fig. 1 – Calculation scheme for logarithmic derivative subject to L-spacing.

See also

Formal science / Mathematics / Calculus

Petroleum Industry / Upstream / Subsurface E&P Disciplines / Well Testing / Pressure Testing / Pressure Transient Analysis (PTA) / PTA Diagnostic Plot / Logarithmic Derivative

References

1. Bourdet, D. "Well Test Analysis: The Use of Advanced Interpretation Models", Elsevier, 2002.
2. Bourdet, D., Ayoub, J.A., and Pirard, Y.M. "Use of Pressure Derivative in Well-Test Interpretation", SPE Formation Evaluation, 1989, June, pp. 293-302.