Derivation of pseudo-linear pressure diffusion @model

We start with reservoir pressure diffusion outside wellbore:

(1) $\frac{\partial(\rho\phi)}{\partial t} + \nabla(\rho\,\mathbf{u}) = 0$	(2) $\int_{\Sigma_k} \mathbf{u}\, d\mathbf{A} = q_k(t)$

where

Σ_k	well-reservoir contact of the k-th well
$d\mathbf{\Sigma}$	normal vector of differential area on the well-reservoir contact, pointing inside wellbore

Then use the following equality:

(3) $d(\rho\,\phi) = \rho\, d\phi + \phi\, d\rho = \rho\,\phi \left(\frac{d\phi}{\phi} + \frac{d\rho}{\rho} \right) = \rho\,\phi \left(\frac{1}{\phi}\frac{d\phi}{dp}\, dp + \frac{1}{\rho}\frac{d\rho}{dp}\, dp \right) = \rho\,\phi\,(c_\phi\, dp + c\, dp) = \rho\,\phi\, c_t\, dp$

to arrive at:

(4) $\rho\,\phi\, c_t \cdot \frac{\partial p}{\partial t} + \nabla(\rho\,\mathbf{u}) = 0$	(5) $\int_{\Sigma_k} \mathbf{u}\, d\mathbf{A} = q_k(t)$

where

$c_t = c_\phi + c$	total compressibility

Let's assume Darcy flow with constant permeability $\frac{dk}{dp} = 0$ and ignore gravity forces:

(6) $\mathbf{u} = \frac{k}{\mu} \nabla p$

so that diffusion equation becomes:

(7) $\rho\,\phi\, c_t \cdot \frac{\partial p}{\partial t} + \nabla(k \cdot \frac{\rho}{\mu} \nabla p) = 0$	(8) $\frac{k}{\mu} \cdot \int_{\Sigma_k} \nabla p \cdot d\mathbf{A} = q_k(t)$

Let's express the density via Z-factor:

(9) $\rho = \frac{M}{RT} \frac{p}{Z(p)}$

where

T	fluid temperature
M	molar mass of a fluid
R	gas constant

and assuming the fluid temperature T does not change over time and space during the modelling period:

(10) $\phi \, c_t \, \mu \cdot \frac{p}{\mu \, Z} \cdot \frac{\partial p}{\partial t} + \nabla \left(k \cdot \frac{p}{\mu \, Z} \nabla p \right) = 0$	(11) $\frac{k}{\mu} \cdot \int_{\Sigma_k} \nabla p \cdot d\mathbf{A} = q_k(t)$

or

(12) $\phi \, c_t \, \mu \cdot \frac{\partial \Psi}{\partial t} + \nabla \left(k \cdot \nabla \Psi \right) = 0$	(13) $\frac{k}{\mu} \cdot \int_{\Sigma_k} \nabla p \cdot d\mathbf{A} = q_k(t)$

where

$\Psi(p) = 2 \int_0^p \frac{p \, dp}{\mu(p) \, Z(p)}$	Pseudo-Pressure

In some practical cases the complex $c_t \, \mu$ can be considered as constant in time which makes (12) a linear differential equation.

But during the early transition times the pressure drop is usually high and the complex $c_t \, \mu$ can not be considered as constant in time which leads to distortion of pressure transient diagnostics at early times.

In this case one can use Pseudo-Time, calculated by means of the bottom-hole pressure $p_{BHP}(t)$:

(14) $$\tau(t) = \int_0^t \frac{dt}{\mu(p_{BHP}) \, c_t(p_{BHP})} \, , \quad p_{BHP} = p_{BHP}(t)$$

to correct early-time transient behaviour which turn equation (12) into:

(15) $$\phi \cdot \frac{\partial \Psi}{\partial \tau} + \nabla \left(k \cdot \nabla \Psi \right) = 0$$

See also

Physics / Mechanics / Continuum mechanics / Fluid Mechanics / Fluid Dynamics / Pressure Diffusion / Pressure Diffusion @model / Pseudo-linear pressure diffusion @model