

Exponential Integral (Ei or E1)

@wikipedia

Two different functions of **real argument** $x \in \mathbb{R}$ are called this way:

(1) $\text{Ei}(x) = - \int_{-x}^{\infty} \frac{e^{-\xi}}{\xi} d\xi$	(2) $\text{E}_1(x) = \int_x^{\infty} \frac{e^{-\xi}}{\xi} d\xi$
---	---

which are related to each other as:

(3)
$$\text{Ei}(x) = -\text{E}_1(-x)$$

There is a trend to moving from Ei definition (which was dominating in the past) towards E_1 which becomes more and more popular nowadays.

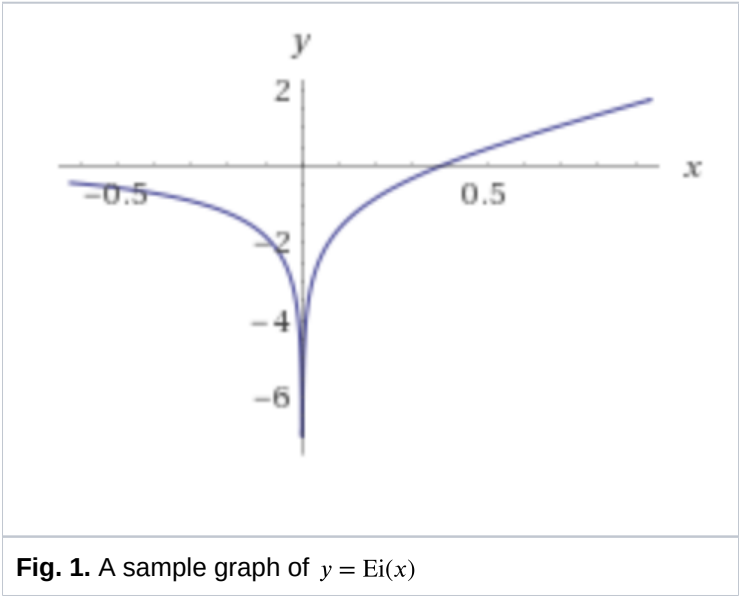


Fig. 1. A sample graph of $y = \text{Ei}(x)$

Properties

(4) $\text{Ei}(0) = -\infty$	(5) $\text{Ei}(-\infty) = 0$	(6) $\text{Ei}(+\infty) = +\infty$	(7) $\frac{d}{dx}\text{Ei}(x) = \frac{e^x}{x}$
------------------------------	------------------------------	------------------------------------	--

Approximations

$ x \ll 1$	$ x \gg 1$
(8) $\operatorname{Ei}(x) = \gamma + \ln x + \sum_{k=1}^{\infty} \frac{x^k}{k \cdot k!}$	(9) $\operatorname{Ei}(x) = e^x \left[\frac{1}{x} + \sum_{k=2}^{\infty} \frac{(k-1)!}{x^k} \right]$
where $\gamma = 0.57721 \dots$ is Euler–Mascheroni constant	
$0 < x \ll 1$	
(10) $\operatorname{Ei}(-x) \sim \gamma + \ln x$	

Application

The [real](#)-value positive function $w(t,r)$ of two [real](#)-value positive arguments ([time](#) t and [radial coordinate](#) r):

(11)
$$w(t,r) = E_1\left(\frac{r^2}{4t}\right) = -\operatorname{Ei}\left(-\frac{r^2}{4t}\right)$$

honours a [planar axial-symmetric diffusion equation](#) with homogenous initial and boundary conditions:

(12) $\frac{\partial w}{\partial t} = \frac{\partial^2 w}{\partial r^2} + \frac{1}{r} \frac{\partial w}{\partial r}$	(13) $w(t = 0, r) = 0$	(14) $w(t, r = \infty) = 0$	(15) $0 \leq w(t, r) < \infty, \forall (t, r) \in D = \{t \geq 0, r > 0\} \subset \mathbb{R}$
--	--------------------------------	-------------------------------------	---

and is widely used in [radial](#) heat-mass transfer analysis.

See also

[Formal science](#) / [Mathematics](#) / [Calculus](#)

References


Unknown macro: 'cite-summary'


Unknown macro: 'single-cite'


Unknown macro: 'single-cite'