

PKN Hydraulic Fracture @model

Fracture half-length	(1) $X_f = \frac{Q}{2 w_f h_f}$
Average Fracture width	(2) $w_f = \frac{2 \pi p_{\text{net}} h_f}{5 E'}$
Net pressure at the wellbore	(3) $p_{\text{net}} = \left(\frac{20}{\pi^2} \cdot \frac{E'^4 q Q \mu}{h_f^6} \right)^{1/5}$

where

t	fracture fluid injection time
q	fracture fluid injection rate
$Q(t) = q \cdot t$	cumulative fracture fluid injection over time t
h_f	fracture height
$E' = E / (1 - \nu^2)$	plane stress
E	Young modulus
ν	Poisson's ratio
μ	fracture fluid viscosity

Fracture half-length	(4) $X_f = 0.524 \left(\frac{q^3 E'}{\mu h_f^4} \right)^{1/5} t^{4/5} = 0.524 \left(\frac{E'}{\mu h_f^4} \frac{Q^4}{q} \right)^{1/5} = 0.8 \cdot \frac{E' Q}{p_{\text{net}} h_f^2} = \frac{Q}{w_f h_f}$
Fracture width at wellbore	(5) $w_{f0} = 3.04 \left(\frac{q^2 \mu}{E' h_f} \right)^{1/5} t^{1/5} = 3.04 \left(\frac{q Q \mu}{E' h_f} \right)^{1/5} = \frac{2 p_{\text{net}} h_f}{E'}$
Average Fracture width	(6) $w_f = \frac{\pi}{5} w_{f0} = \frac{2\pi p_{\text{net}} h_f}{5 E'}$
Net pressure at the wellbore	(7) $p_{\text{net}} = 1.524 \left(\frac{E'^4 q^2 \mu}{h_f^6} \right)^{1/5} t^{1/5} = 1.524 \left(\frac{E'^4 q Q \mu}{h_f^6} \right)^{1/5}$

where

$t = Q(t)/q$	injection time
q	injection rate
$Q(t)$	cumulative injection over time t
h_f	fracture height
$E' = \frac{E}{1-\nu^2}$	plain stress
E	Young modulus
ν	Poisson ratio
μ	fluid viscosity

See Also

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Reference

Perkins, Kern and Nordgren