Pseudo-linear pressure diffusion @model

(1) $\phi \cdot \frac{\partial \Psi}{\partial \tau} - \nabla \cdot \left(k \cdot \vec{\nabla} \Psi \right) = 0$	(2) $-\frac{k}{\mu} \int_\Sigma \nabla p \, d\mathbf{\Sigma} = q(t)$

where

$p(t, \mathbf{r})$	reservoir pressure	t	time
$\phi(\mathbf{r})$	effective porosity	$\mathbf{r}$	position vector
$c_t(p)$	total compressibility	∇	gradient operator
k	formation permeability to a given fl uid	$d\mathbf{\Sigma}$	normal surface element of well-reservoir contact
$\mu(p)$	dynamic viscosity of a given fluid	$\Psi(p) = 2 \int_0^p \frac{p \, dp}{\mu(p) \, Z(p)}$	Pseudo-Pressure
$Z(p)$	fluid compressibility factor	$\tau(t) = \int_0^t \frac{dt}{\mu(p) \, c_t(p)}$	Pseudo-Time
		$q(t)$	sandface flowrates to/from the well-reservoir contact

Derivation of pseudo-linear pressure diffusion @model

In some practical cases the complex $c_t \, \mu$ can be considered as constant in time which makes Pseudo-Time being proportional to freagular time:

(3) $$\tau(t) = \frac{t}{\mu \, c_t}$$

and one can write the diffusion equation as:

(4) $$\phi \, c_t \, \mu \cdot \frac{\partial \Psi}{\partial \tau} - \nabla \cdot \left(k \cdot \vec{\nabla} \Psi \right) = 0$$

which is a treat it as a differential equation with linear coefficients.

But during the early transition times the pressure drop is usually high and the complex $c_t \, \mu$ can not be considered as constant in time which leads to distortion of pressure transient diagnostics at early times.

In this case one can use Pseudo-Time, calculated by means of the bottom-hole pressure $p_{BHP}(t)$:

(5) $$\tau(t) = \int_0^t \frac{dt}{\mu(p_{BHP}(t)) \, c_t(p_{BHP}(t))}$$

to correct early-time transient behaviour.

In case of the ideal gas equation of state, the Z-factor has a unit value: $Z(p) = 1$, viscosity does not depend on pressure $\mu(p) = \mu$ and total compressibility is fully defined by fluid compressibility $c_t = c_r + c \sim \frac{1}{p}$ which simplifies the expression for Pseudo-Pressure and Pseudo-Time as to:

(6)	$\Psi(p) = \frac{p^2}{\mu}$	(7)	$\tau(t) = \frac{1}{\mu} \int_0^t p_{BHP}(t) dt$

See also

Physics / Mechanics / Continuum mechanics / Fluid Mechanics / Fluid Dynamics / Pressure Diffusion / Pressure Diffusion @model