

Pseudo-linear pressure diffusion @model

(1) $\phi \cdot \frac{\partial \Psi}{\partial \tau} - \nabla \cdot (k \cdot \vec{\nabla} \Psi) = 0$	(2) $-\frac{k}{\mu} \int_{\Sigma} \nabla p d\Sigma = q(t)$
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where

$p(t, \mathbf{r})$	reservoir pressure	t	time
$\phi(\mathbf{r})$	effective porosity	$\mathbf{r}$	position vector
$c_t(p)$	total compressibility	∇	gradient operator
k	formation permeability to a given fluid	$d\Sigma$	normal surface element of well-reservoir contact
$\mu(p)$	dynamic viscosity of a given fluid	$\Psi(p) = 2 \int_0^p \frac{p dp}{\mu(p) Z(p)}$	Pseudo-Pressure
$Z(p)$	fluid compressibility factor	$\tau(t) = \int_0^t \frac{dt}{\mu(p) c_t(p)}$	Pseudo-Time
		$q(t)$	sandface flowrates to/from the well-reservoir contact

Derivation of pseudo-linear pressure diffusion @model

In some practical cases the complex $c_t \mu$ can be considered as constant in time which makes [Pseudo-Time](#) being proportional to feagular time:

$$(3) \quad \tau(t) = \frac{t}{\mu c_t}$$

and one can write the diffusion equation as:

$$(4) \quad \phi c_t \mu \cdot \frac{\partial \Psi}{\partial \tau} - \nabla \cdot (k \cdot \vec{\nabla} \Psi) = 0$$

which is a treat it as a differential equation with linear coefficients.

But during the early transition times the pressure drop is usually high and the complex $c_t \mu$ can not be considered as constant in time which leads to distortion of [pressure transient diagnostics](#) at early times.

In this case one can use [Pseudo-Time](#), calculated by means of the [bottom-hole pressure](#) $p_{BHP}(t)$:

$$(5) \quad \tau(t) = \int_0^t \frac{dt}{\mu(p_{BHP}(t)) c_t(p_{BHP}(t))}$$

to correct [early-time transient](#) behaviour.

In case of the ideal gas equation of state, the [Z-factor](#) has a unit value: $Z(p) = 1$, viscosity does not depend on pressure $\mu(p) = \mu$ and total compressibility is fully defined by fluid compressibility $c_t = c_r + c \sim \frac{1}{p}$ which simplifies the expression for [Pseudo-Pressure](#) and [Pseudo-Time](#) as to:

$(6) \qquad \Psi(p) = \frac{p^2}{\mu}$	$(7) \qquad \tau(t) = \frac{1}{\mu} \int_0^t p_{BHP}(t) dt$
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See also

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