Derivation of Pressure Profile in G-Proxy Pipe Flow @model

Let's start with Pressure Profile in Homogeneous Steady-State Pipe Flow @model:

(1) $\left[\rho(p) - j_m^2 \cdot c(p) \right] \frac{dp}{dl} = \rho^2(p) \, g \, \cos\theta(l) - \frac{j_m^2}{2d} \cdot f(p)$	(2) $p(l=0) = p_0$

and assume constant pipe inclination:

(3) $$\theta(l) = \theta = \mathrm{const}$$

Let's define constant number:

(4) $$G = g \cdot \cos\theta = \mathrm{const}$$

and rewrite the equation (1) as:

(5) $$\frac{\left[\rho(p) - j_m^2 \cdot c(p) \right] \, dp}{\rho^2(p) \, G - \frac{j_m^2}{2d} \cdot f(p)} = dl$$

The integration of the left side of (5) with the boundary condition (2) leads to:

(6) $$L = \int_{p_0}^{p} \frac{\rho(p) - j_m^2 \, c(p)}{G \, \rho^2(p) - F(\rho(p))} \, dp$$

where

$$\boxed{F(\rho) = \frac{j_m^2}{2d} \cdot f(\rho)}$$

This can be further re-written as:

(7) $$L = \int_{p_0}^{p} \frac{\rho \, dp}{G \, \rho^2 - F(\rho)} - j_m^2 \cdot \int_{\rho_0}^{\rho} \frac{1}{\rho} \, \frac{d\rho}{G \, \rho^2 - F(\rho)}$$

or

(8) $$L = \int_{\rho_0}^{\rho} \frac{1/c(\rho) - j_m^2/\rho}{G \, \rho^2 - F(\rho)} \, d\rho$$

See also

Physics / Fluid Dynamics / Pipe Flow Dynamics / Pipe Flow Simulation / Pressure Profile in Homogeneous Steady-State Pipe Flow @model / Pressure Profile in G-Proxy Pipe Flow @model