Multiphase Pipe Flow

Outputs

$\{s_\alpha\}_{\alpha=1..n}$	phase holdup
$\{q_\alpha\}_{\alpha=1..n}$	phase volumetric flowrate

Inputs

A	pipe cross-sectional area
$\{\dot m_\alpha\}_{\alpha=1..n}$	phase mass flowrates
$\{\rho_\alpha\}_{\alpha=1..n}$	phase densities

Solver

(1)	$s_\alpha = \frac{\dot m_\alpha}{\rho_\alpha \, u_\alpha} \cdot \left(\sum_\beta \frac{\dot m_\beta}{\rho_\beta \, u_\beta} \right)^{-1}$
(2)	$q_\alpha = s_\alpha \, u_\alpha \, A$

Derivation

Given the multiphase flow of n phases: $\alpha = 1..n$ and mass flowrates $\dot m_\alpha$

(3) $$\dot m = \sum_\alpha \dot m_\alpha$$

(4) $$A = \sum_\alpha A_\alpha$$

(5) $$s_\alpha = A_\alpha / A$$

(6) $$\sum_\alpha s_\alpha = 1$$

(7) $$u_m = \sum_\alpha s_\alpha \cdot \dot u_\alpha$$

(8) $$q_\alpha = \dot m_\alpha / \rho_\alpha = A_\alpha \, u_\alpha \Rightarrow \dot m_\alpha = \rho_\alpha \, A_\alpha \, u_\alpha$$

For homogeneous pipe flow: $u_\alpha = u_m, \forall \alpha \in [1..n]$ and volumetric shares are going to be:

(9) $$s_\alpha = \frac{\dot m_\alpha}{\rho_\alpha} \cdot \left(\sum_\beta \frac{\dot m_\beta}{\rho_\beta} \right)^{-1}$$

See also