Derivation of linear single-phase pressure diffusion @model

We start with the general from of (Single-phase pressure diffusion @model:1):

(1) $\phi \cdot c_t \cdot \partial_t p + \nabla \mathbf{u} + c \cdot (\mathbf{u} \, \nabla p) = \sum_k q_k(t) \cdot \delta(\mathbf{r} - \mathbf{r}_k)$	(2) $\mathbf{u} = - M \cdot (\nabla p - \rho \, \mathbf{g})$	(3) $\int_\Gamma \mathbf{u} \, d\mathbf{\Sigma} = q_\Gamma(t)$

where

$p(t, \mathbf{r})$	reservoir pressure	t	time
$\rho(\mathbf{r})$	fluid density	$\mathbf{r}$	position vector
$\phi(\mathbf{r})$	effective porosity	$\mathbf{r}_k$	position vector of the k-th source
$c_t(\mathbf{r})$	total compressibility	$\delta(\mathbf{r})$	Dirac delta function
$M(\mathbf{r})$	reservoir fluid mobility $M(\mathbf{r}) = \frac{k(\mathbf{r})}{\mu}$	∇	gradient operator
$k(\mathbf{r})$	formation permeability to a given fluid	$\mathbf{g}$	gravity vector
μ	dynamic viscosity of a given fluid	$\mathbf{u}$	fluid velocity under Darcy flow
$q_k(t)$	sandface flowrates of the k-th source	Γ	reservoir boundary
$q_\Gamma(t)$	flow through the reservoir boundary Γ, which is aquifer or gas cap		

Let's neglect the non-linear term $c \cdot (\mathbf{u} \, \nabla p)$ for low compressibility fluid $c \sim 0$ which is equivalent to assumption of nearly constant fluid density: $\rho(p) = \rho = \mathrm{const}$.

Together with constant pore compressibility $c_\phi = \mathrm{const}$ this will lead to constant total compressibility $c_t = c_\phi + c \approx \mathrm{const}$.

Assuming that permeability and fluid viscosity do not depend on pressure $k(p) = k = \mathrm{const}$ and $\mu(p) = \mu = \mathrm{const}$ one arrives to the differential equation with constant coefficients:

(4) $\phi \cdot c_t \cdot \partial_t p + \nabla \mathbf{u} = \sum_k q_k(t) \cdot \delta(\mathbf{r} - \mathbf{r}_k)$	(5) $\mathbf{u} = - M \cdot (\nabla p - \rho \, \mathbf{g})$	(6) $\int_\Gamma \mathbf{u} \, d\mathbf{\Sigma} = q_\Gamma(t)$	
or			
(7) $\phi \cdot c_t \cdot \partial_t p + \nabla \mathbf{u} = 0$	(8) $\mathbf{u} = - M \cdot (\nabla p - \rho \, \mathbf{g})$	(9) $\int_{\Sigma_k} \mathbf{u} \, d\mathbf{\Sigma} = q_k(t)$	(10) $\int_\Gamma \mathbf{u} \, d\mathbf{\Sigma} = q_\Gamma(t)$

See also

Physics / Mechanics / Continuum mechanics / Fluid Mechanics / Fluid Dynamics / Pressure Diffusion / Pressure Diffusion @model / Single-phase pressure diffusion @model