Compressibility factor

@wikipedia

Synonyms: Compressibility factor = Z-factor

Disclaimer: Not to be confused with Compressibility c.

Dimensionless multiplier describing the deviation of a fluid density from ideal gas estimate under the same pressure & temperature conditions:

(1) $$Z = \frac{p \, V_m}{R \, T} = \frac{p}{\rho} \cdot \frac{M}{R \, T}$$

where

p	fluid pressure	$V_m = V/\nu$	fluid molar volume
T	fluid temperature	V	fluid volume
ρ	fluid density	ν	amount of substance
R	gas constant	M	molar mass of a fluid

Alternatively Z-factor can be expressed through the dynamic fluid properties at reference conditions as:

(2) $$Z(T, p) = Z^\circ \cdot \frac{\rho^\circ \, T^\circ}{p^\circ} \cdot \frac{p}{\rho(T, p) \, T}$$

where $()^\circ$ means reference conditions, usually Standard Conditions (STP).

Z-factor can be used to calculate fluid density ρ and Formation Volume Factor (FVF) B as:

(3) $\rho(T, p) = \rho^\circ \cdot \frac{Z^\circ \, T^\circ}{p^\circ} \cdot \frac{p}{Z(T, p) \, T}$	(4) $B(T, p) = \frac{\rho^\circ}{\rho(T, p)} = \frac{p^\circ}{Z^\circ \, T^\circ} \cdot \frac{Z(T, p) \, T}{p}$

Z-factor is related to fluid compressibility c as:

(5) $c(p) = \frac{1}{p} - \frac{1}{Z} \frac{dZ}{dp}$	(6) $Z(p) = Z_0 \cdot \frac{p}{p_0} \cdot \exp \left[- \int_{p_0}^{p} c(p) dp \right]$

(7) $c = \frac{1}{\rho}\frac{d\rho}{dp} = \frac{d\ln\rho}{dp} = \frac{d}{dp}\left(\ln\left(\frac{p}{Z}\right)\right) = \frac{Z}{p}\cdot\frac{d}{dp}\left(\frac{p}{Z}\right) = \frac{Z}{p}\cdot\left(\frac{1}{Z} + p\cdot\frac{d}{dp}\left(\frac{1}{Z}\right)\right) = \frac{1}{p} - \frac{1}{Z}\frac{dZ}{dp}$

Rewriting (5):

(8) $\frac{d\ln Z}{dp} = \frac{1}{p} - c(p) \rightarrow \ln\frac{Z}{Z_0} = \ln\frac{p}{p_0} - \int_{p_0}^{p} c(p)\,dp$

which arrives to (6).

The Z-factor value for Ideal Gas is strictly unit: $Z(T,p) = 1$.

For many real gases (particularly for the most compositions of natural gases) the Z-factor is trending towards unit value ($Z \rightarrow 1$) while approaching the STP.

For incompressible fluids the Z-factor is trending to linear pressure dependence ($Z \rightarrow a\cdot p$) with pressure growth.

Modelling Z-factor $Z(T,p)$ as a function of fluid pressure p and temperature T is based on Equation of State.

There is also a good number of explicit Z-factor Correlations @models.

See also

Natural Science / Physics / Thermodynamics / Equation of State

[Compressibility][Fluid Compressibility][Gas compressibility]

References