Derivation of Pressure Profile in Steady-State Homogeneous Pipe Flow @model

The base driving equations of a pipe flow are:

Steady-state 1D inviscid fluid flow	Pipe Flow Mass Conservation
(1) $\frac{dp}{dl} = -\rho\, u \frac{du}{dl} + \rho\, g\, \cos\theta + f_{\rm cnt,\, l}$	(2) $j_m(l) = j_m = \rho(l) \cdot u = \rm const$
Equation of State (EOS)	**Darcy–Weisbach**
(3) $\rho = \rho(p, T)$	(4) $f_{\rm cnt,l} = -f \cdot \frac{\rho\, u^2}{2d}$

where

l	distance along the fluid flow streamline
$\theta(l)$	inclinational deviation, $\cos\theta = dz/dl$
$z(l)$	elevation along the 1D flow trajectory
$T(l)$	fluid temperature
$p(l)$	fluid pressure
$\rho(l)$	fluid density
$\mathbf{u}(l)$	fluid velocity vector
$u(l)$	superficial velocity of the pipe flow
$\mathbf{f}_{\rm cnt}(l)$	volumetric density of all contact forces exerted on fluid body
$f_{\rm cnt,l}(l) = \mathbf{e}_u \cdot \mathbf{f}_{\rm cnt}$	projection of $\mathbf{f}_{\rm cnt}$ onto the unit fluid velocity vector: $\mathbf{e}_u = \lvert\mathbf{u}\rvert^{-1}\, \mathbf{u}$
$j_m = \rho(l) \cdot u(l)$	fluid mass flux
$\dot m$	mass flowrate
g	standard gravity constant

Substituting (2) and (4) into (1):

(5) $$\frac{dp}{dl} = -j_m \cdot \frac{d}{dl}\left(\frac{j_m}{\rho}\right) + \rho\, g\, \cos\theta - f \cdot \frac{\rho}{2d} \cdot \left(\frac{j_m}{\rho}\right)^2$$

(6) $$\frac{dp}{dl} = j_m^2 \cdot \frac{1}{\rho^2} \frac{d\rho}{dl} + \rho\, g\, \cos\theta - \frac{j_m^2}{2d} \cdot \frac{f}{\rho}$$

(7) $$\frac{dp}{dl} = j_m^2 \cdot \frac{1}{\rho^2} \frac{d\rho}{dp} \cdot \frac{dp}{dl} + \rho\, g\, \cos\theta - \frac{j_m^2}{2d} \cdot \frac{f}{\rho}$$

(8) $$\frac{dp}{dl} = j_m^2 \cdot \frac{1}{\rho} \cdot c \cdot \frac{dp}{dl} + \rho\, g\, \cos\theta - \frac{j_m^2}{2d} \cdot \frac{f}{\rho}$$

and finally

(9) $$\left(1 - j_m^2 \cdot \frac{c}{\rho}\right) \frac{dp}{dl} = \rho \, g \, \cos\theta - \frac{j_m^2}{2d} \cdot \frac{f}{\rho}$$

Alternative forms

(10) $\left[\rho - j_m^2 \, c\right] \cdot \frac{dp}{dl} = \rho^2 \, g \, \cos\theta - \frac{j_m^2}{2d} \cdot f(\rho)$	(11) $\left[\frac{1}{c} - \frac{j_m^2}{\rho}\right] \cdot \frac{d\rho}{dl} = \rho^2 \, g \, \cos\theta - \frac{j_m^2}{2d} \cdot f(\rho)$

See Also

Petroleum Industry / Upstream / Pipe Flow Simulation / Water Pipe Flow @model / Stationary Isothermal Homogenous Pipe Flow Pressure Profile @model

[Darcy friction factor] [Darcy friction factor @model]

[Euler equation]