Darcy friction factor in water producing/injecting wells @model

Motivation

Assess how Darcy friction factor is varying along the flow path of water producing/injecting wells

Conclusion

In many engineering applications the Darcy friction factor in stationary water flow in a constant diameter pipe can be approximated as a constant along hole: $f(l) = f_s = \mathrm{const}$ with absolute value depending on the flowrate q.

The along-hole variation Darcy friction factor is usually not exceeding 10 % but the contribution of the friction-based pressure loss to the gravity-based pressure build up in vertical and slanted wells is very minor (few percents only) which makes constant friction factor assumption quite relevant.

The absolute value is staying between $f = 0.04$ for the very small flow rates (< 100 cmd in 3" pipe) and $f = 0.015$ for the very high flow rates (> 1,000 cmd in 3" pipe) which makes a substantial difference in slanted and horizontal wells.

For complex well designs with varying pipe flow diameters and water source/stocks which may lead to substantial variation of flowrate the wellbore model can be split in segments each having a constant friction factor.

Derivation

Consider a ratio of along-hole friction-based pressure drop $\left[\frac{dp}{dl} \right]_f = \frac{\rho_s \, q_s^2}{2 A^2 d} f_s$ and gravity-based pressure drop in vertical well $\left[\frac{dp}{dl} \right]_g = \rho_s \, g$:

(1) $$\frac{[dp/dl]_f}{[dp/dl]_g} = \frac{q_s^2}{2 A^2 \cdot d \cdot g} f_s = \frac{f_s \, u_s^2}{2 \cdot d \cdot g}$$

In 3" tubing with high flowrate (500 m3/d) the flow velocity is going to be around 1.3 m/s and the ratio (1) is going to be $\frac{[dp/dl]_f}{[dp/dl]_g} \sim 3.3\%$.

Furthermore, Darcy friction factor f for wellbore flow can be written as:

(2) $$\mathrm{Re}(l) = \frac{u(l) \cdot d}{\nu(l)} = \frac{4 \rho_s q_s}{\pi d} \frac{1}{\mu(T, p)}$$

The along-hole variation of Darcy friction factor f is due to the influence of pressure $p(l)$ and temperature $T(l)$ variations on the fluid viscosity $\mu(T, p)$.

In vertical and slanted wells both temperature and pressure are growing with depth.

The decrease in water viscosity with growing temperature is partially compensated by decrease in response to growing pressure thus making viscosity staying within 10% along-hole in most practical cases (usually slightly decreasing with depth).

Providing that friction losses are only 3.3 % of the hydrostatic column the further variation of Darcy friction factor by 10% provides only 0.33 % error against pressure modelling with constant Darcy friction factor.

In case of slanted wells even a strong inclination will not change the friction contribution by much (may see a slight increase from 3 % up to 5 %).

For the horizontal sections of wells and surface pipelines the value of friction-based pressure loss dominates over vanishing gravity-based pressure build up which zooms the value of accurate calculation of Darcy friction factor with account of its variation along the flow. In the meantime, for strongly inclined/horizontal pipelines the pressure /temperature variation along the pipe is usually very minor, so is the water viscosity, and Darcy friction factor again has very little variation along the flow.

See also

Physics / Fluid Dynamics / Pipe Flow Dynamics / Darcy–Weisbach equation / Darcy friction factor

[Fluid friction with pipeline walls][Darcy friction factor in water producing/injecting wells @model]