Steady State Radial Flow Pressure Diffusion @model

Motivation

In many practical cases the Radial Flow Pressure Diffusion is evolving towards pressure stabilization and can be efficiently analyzed using the steady state flow model.

Inputs & Outputs

Inputs		Outputs	
q_t	total sandface rate	$p(r)$	reservoir pressure
p_i	initial formation pressure	p_{wf}	well bottomhole pressure
σ	transmissibility, $\sigma = \frac{k\,h}{\mu}$		
S	skin-factor		
r_w	wellbore radius		
r_e	drainage radius		

k	absolute permeability
h	effective thickness
μ	dynamic fluid viscosity
ϕ	porosity

Physical Model

Radial fluid flow	Homogenous reservoir	Finite reservoir flow boundary	Slightly compressible fluid flow	Constant rate	Consta skin
$p(t, \mathbf{r}) \rightarrow p(r)$ $\mathbf{r} \in \mathbb{R}^2 = \{x, y\}$	$M(r,p) = M = \text{const}$ $\phi(r,p) = \phi = \text{const}$ $h(r) = h = \text{const}$ $c_r(r) = c_r = \text{const}$	$r_w \leq r \leq r_e < \infty$	$c_t(r,p) = \text{const}$	$q_t = \text{const}$	$S =$

Mathematical Model

(1) $r_{wf} < r \le r_e$ (2) $p(t,r) = p(r) \Leftrightarrow \frac{\partial p}{\partial t} = 0$	(3) $\frac{\partial^2 p}{\partial r^2} + \frac{1}{r}\frac{\partial p}{\partial r} = 0$	(4) $p(r_e) = p_i$	(5) $\left[r \frac{\partial p(r)}{\partial r} \right]_{r \to r_w} = \frac{q_t}{2\pi\sigma}$
(6) $p_{wf} = p(r_w) - S \cdot r_w \left. \frac{\partial p}{\partial r} \right\|_{r=r_w}$			
(7) $p(r) = p_i + \frac{q_t}{2\pi\sigma} \ln \frac{r}{r_e} = p(r_w) + \frac{q_t}{2\pi\sigma} \ln \frac{r}{r_w}, \quad r_{wf} < r \le r_e$		(8) $p_{wf} = p_i - \frac{q_t}{2\pi\sigma} \left[S + \ln \frac{r_e}{r_w} \right]$	

Applications

Equation (7) shows how the basic diffusion model parameters impact the relation between drawdown $\Delta p = p_i - p_{wf}$ and total sandface flowrate q_t and plays important methodological role as they are used in many algorithms and express-methods of Pressure Testing.

The Total Sandface Productivity Index for low-compressibility fluid and low-compressibility rocks does not depend on formation pressure, bottomhole pressure and the flowrate and can be expressed as:

(9) $$J_t = \frac{q_t}{p_i - p_{wf}(t)} = \frac{2\pi\sigma}{\ln \frac{r_e}{r_w} + S} = \text{const}$$

The Field-average Productivity Index for low-compressibility fluid and low-compressibility rocks does not depend on formation pressure, bottomhole pressure and the flowrate and can be expressed as:

(10) $$J_t = \frac{q_t}{p_r(t) - p_{wf}(t)} = \frac{2\pi\sigma}{\ln \frac{r_e}{r_w} + 0.5 + S} = \text{const}$$

See Also

Physics / Mechanics / Continuum mechanics / Fluid Mechanics / Fluid Dynamics / Radial fluid flow / Pressure diffusion / Pressure Diffusion @model / Radial Flow Pressure Diffusion @model

Petroleum Industry / Upstream / Subsurface E&P Disciplines / Well Testing / Pressure Testing

[Well & Reservoir Surveillance]