Pressure Line Source Solution (LSS) @model

Motivation

Reservoir pressure dynamics away from wellbore and boundaries is sensitive to the two specific complex reservoir properties: transmissibility σ and pressure diffusivity χ.

In case the reservoir flow has been created by a well (vertical or horizontal) it will trend to form a radial flow away from boundaries and well itself.

In this case a pressure drop and well flowrate can be roughly related to each other by means of a simple analytical homogeneous reservoir flow model with wellbore and boundary effects neglected.

Since the well radius is neglected the well is modeled as a vertical 0-thickness line, sourcing the fluid from a reservoir, giving a model a specific name Line Source Solution.

Inputs & Outputs

Inputs		Outputs	
q_t	total sandface rate	$p(t,r)$	reservoir pressure
p_i	initial formation pressure		
σ	transmissibility		
χ	pressure diffusivity		

$\sigma = \frac{k\,h}{\mu}$	transmissibility		μ	dynamic fluid viscosity
$\chi = \frac{k}{\mu}\,\frac{1}{\phi\,c_t}$	pressure diffusivity		t	time
k	absolute permeability		r	radial direction
ϕ	porosity		c_t	total compressibility

Physical Model

Radial fluid flow	**Homogenous reservoir**	**Infinite boundary**	**Zero wellbore radius**	**Slightly compressible fluid flow**	**Constant rate production**

$p(t,r)$	$M(r,p) = M = \text{const}$ $\phi(r,p) = \phi = \text{const}$ $h(r) = h = \text{const}$ $c_r(r) = c_r = \text{const}$	$r \to \infty$	$r_w = 0$	$c_t(p) = c_r + c = \text{const}$	$q_t = \text{const}$

Mathematical Model

Motion equation	Initial condition	Boundary conditions	
(1) $\frac{\partial p}{\partial t} = \chi \left[\frac{\partial^2 p}{\partial r^2} + \frac{1}{r} \frac{\partial p}{\partial r} \right]$	(2) $p(t=0,r) = p_i$	(3) $p(t, r=\infty) = p_i$	(4) $\left[r \frac{\partial p}{\partial r} \right]_{r=0} = \frac{q_t}{2\pi\sigma}$

Computational Model

(5) $p(t,r) = p_i + \frac{q_t}{4\pi\sigma} \text{Ei} \left(-\frac{r^2}{4\chi t} \right)$	$\text{Ei}(\xi)$ – exponential integral

Approximations

Infinite Acting Radial Flow(IARF)
$t \gg \frac{r^2}{4\chi}$
(6) $p(t,r) \sim p_i + \frac{q_t}{4\pi\sigma} \left[\gamma + \ln \left(\frac{r^2}{4\chi t} \right) \right] = p_i - \frac{q_t}{4\pi\sigma} \ln \left(\frac{2.24585\ \chi t}{r^2} \right)$

Diagnostic Plots

	Pressure Drop
	(7) $\delta p = p_i - p_{wf}(t) = \frac{q_t}{4\pi\sigma} \text{Ei} \left(-\frac{r}{4.} \right.$

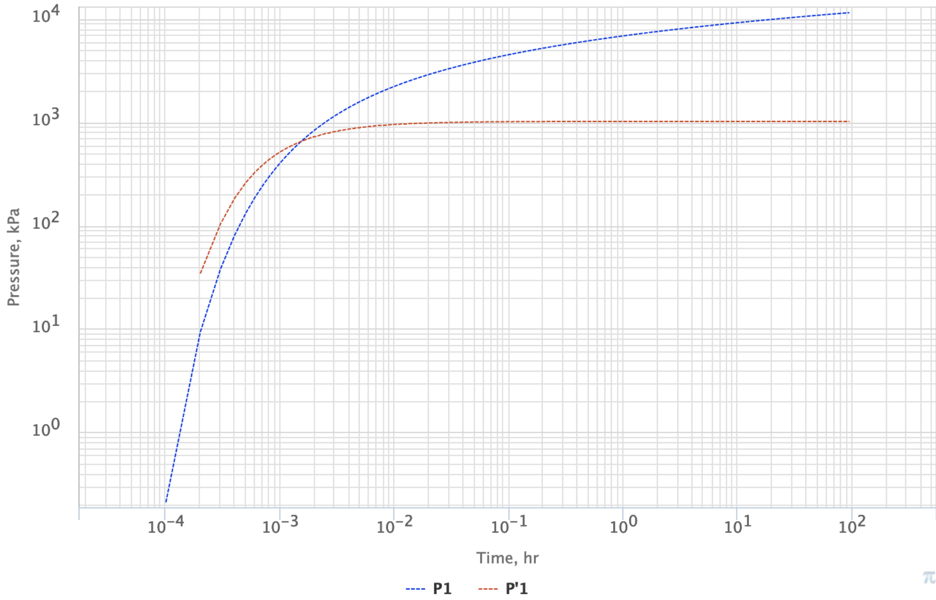

Fig. 1. PTA Diagnostic Plot for LSS pressure response for the 0.1 md reservoir in a close line source vicinity (0.1 m), which is about a typical wellbore size.

One can easily see that with wellbore effects neglected even for a very low permeability reservoir the IARF regime is getting formed very early at 0.01 hr (36 s).

$$\delta p \sim \ln t + \text{const}, \ t \gg \frac{r^2}{4\chi} \tag{9}$$

See also

Physics / Fluid Dynamics / Radial fluid flow

[Radial Flow Pressure @model] [1DR pressure diffusion of low-compressibility fluid] [Exponential Integral]