

Exponential Integral (Ei or E1)

@wikipedia

Two different functions of [real argument](#) $x \in \mathbb{R}$ are called this way:

$$(1) \quad \text{Ei}(x) = - \int_{-x}^{\infty} \frac{e^{-\xi}}{\xi} d\xi$$

$$(2) \quad \text{E}_1(x) = \int_x^{\infty} \frac{e^{-\xi}}{\xi} d\xi$$

which are related to each other as:

$$(3) \quad \text{Ei}(x) = -\text{E}_1(-x)$$

There is a trend to moving from Ei definition (which was dominating in the past) towards E_1 which becomes more and more popular nowdays.

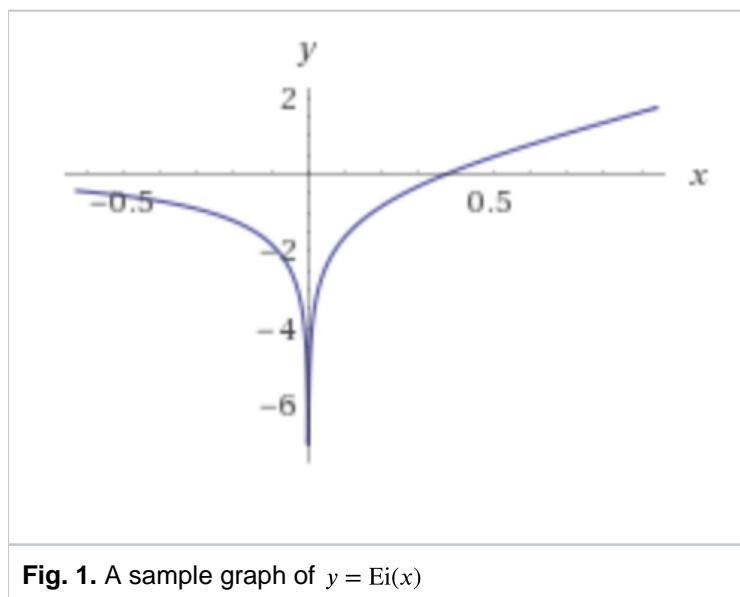


Fig. 1. A sample graph of $y = \text{Ei}(x)$

Properties

$$(4) \quad \text{Ei}(0) = -\infty$$

$$(5) \quad \text{Ei}(-\infty) = 0$$

$$(6) \quad \text{Ei}(+\infty) = +\infty$$

$$(7) \quad \frac{d}{dx} \text{Ei}(x) = \frac{e^x}{x}$$

Approximations

$ x \ll 1$	$ x \gg 1$
(8) $\text{Ei}(x) = \gamma + \ln x + \sum_{k=1}^{\infty} \frac{x^k}{k \cdot k!}$	(9) $\text{Ei}(x) = e^x \left[\frac{1}{x} + \sum_{k=2}^{\infty} \frac{(k-1)!}{x^k} \right]$
where $\gamma = 0.57721 \dots$ is Euler–Mascheroni constant	
$0 < x \ll 1$	
(10) $\text{Ei}(-x) \sim \gamma + \ln x$	

Application

The real-value positive function $w(t, r)$ of two real-value positive arguments (time t and radial coordinate r):

$$(11) \quad w(t, r) = E_1\left(\frac{r^2}{4t}\right) = -\text{Ei}\left(-\frac{r^2}{4t}\right)$$

honours a planar axial-symmetric diffusion equation with homogenous initial and boundary conditions:

(12) $\frac{\partial w}{\partial t} = \frac{\partial^2 w}{\partial r^2} + \frac{1}{r} \frac{\partial w}{\partial r}$	(13) $w(t=0, r) = 0$	(14) $w(t, r=\infty) = 0$	(15) $0 \leq w(t, r) < \infty, \forall (t, r) \in D = \{t \geq 0, r > 0\} \subset \mathbb{R}$
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and is widely used in radial heat-mass transfer analysis.

See also

[Formal science](#) / [Mathematics](#) / [Calculus](#)

References

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