Exponential Integral (Ei or E1)

@wikipedia

Two different functions of real argument $x \in \mathbb{R}$ are called this way:

(1) $\mathrm{Ei}(x) = -\int_{-x}^{\infty} \frac{e^{-\xi}}{\xi}\, d\xi$	(2) $\mathrm{E}_1(x) = \int_{x}^{\infty} \frac{e^{-\xi}}{\xi}\, d\xi$

which are related to each other as:

(3) $\mathrm{Ei}(x) = -E_1(-x)$

There is a trend to moving from Ei definition (which was dominating in the past) towards E_1 which becomes more and more popular nowdays.

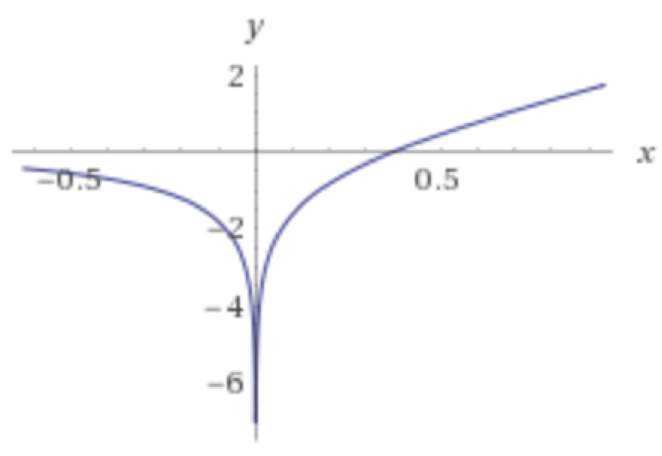

Fig. 1. A sample graph of $y = \mathrm{Ei}(x)$

Properties

(4) $\mathrm{Ei}(0) = -\infty$	(5) $\mathrm{Ei}(-\infty) = 0$	(6) $\mathrm{Ei}(+\infty) = +\infty$	(7) $\frac{d}{dx}\mathrm{Ei}(x) = \frac{e^x}{x}$

Approximations

$\lvert x\rvert \ll 1$	$\lvert x\rvert \gg 1$
(8) $\mathrm{Ei}(x) = \gamma + \ln \lvert x\rvert + \sum_{k=1}^{\infty} \frac{x^k}{k \cdot k!}$	(9) $\mathrm{Ei}(x) = e^x \left[\frac{1}{x} + \sum_{k=2}^{\infty} \frac{(k-1)!}{x^k} \right]$
where $\gamma = 0.57721\ldots$ is Euler–Mascheroni constant	
$0 < x \ll 1$	
(10) $\mathrm{Ei}(-x) \sim \gamma + \ln x$	

Application

The real-value positive function $\mathrm{w}(t,r)$ of two real-value positive arguments (time t and radial coordinate r):

(11) $$\mathrm{w}(t,r) = E_1\left(\frac{r^2}{4t}\right) = -\mathrm{Ei}\left(-\frac{r^2}{4t}\right)$$

honours a planar axial-symmetric diffusion equation with homogenous initial and boundary conditions:

(12) $\frac{\partial \mathrm{w}}{\partial t} = \frac{\partial^2 \mathrm{w}}{\partial r^2} + \frac{1}{r}\frac{\partial \mathrm{w}}{\partial r}$	(13) $\mathrm{w}(t=0,r) = 0$	(14) $\mathrm{w}(t, r=\infty) = 0$	(15) $0 <= \mathrm{w}(t,r) < \infty\,,\ \forall (t,r) \in D = \{t \geq 0, r > 0\} \subset \mathbb{R}$

and is widely used in radial heat-mass transfer analysis.

See also

Formal science / Mathematics / Calculus

References

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