Continuity equation @model

@wikipedia

Mathematical form of Mass Conservation for continuum body:

Integral form	Differential form
(1) $\frac{d}{dt} \iiint_\Omega \rho \, dV = \frac{dm_\Omega}{dt}$	(2) $\frac{\partial \rho}{\partial t} + \nabla(\rho \, \mathbf{u}) = \frac{d\rho(t,\mathbf{r})}{dt}$

where

t	time	$\rho(t,\mathbf{r})$	continuum body spatial density distribution
$\mathbf{r}$	position vector	$\mathbf{u}(t,\mathbf{r})$	continuum body spatial velocity distribution
Ω	space volume (could be finite or infinite)	$\frac{dm_\Omega}{dt}$	mass generation rate with the space volume Ω
∇	gradient operator	$\frac{d\rho(t,\mathbf{r})}{dt}$	volume-specific mass generation rate at a given point in space $\mathbf{r}$

For the specific case of stationary process when density is not explicitly dependent on time:

(3) $$\frac{\partial \rho}{\partial t} = 0 \rightarrow \nabla(\rho \, \mathbf{u}) = 0$$

For the specific case of finite number of mass generation locations the differential equation (2) takes form:

(4) $$\frac{\partial \rho}{\partial t} + \nabla(\rho \, \mathbf{u}) = \sum_k \dot{m}_k(t) \cdot \delta(\mathbf{r} - \mathbf{r}_k)$$

where

$\mathbf{r}_k$	position vector of the k-th source/sink
$\dot{m}_k(t)$	mass generation rate at k-th source/sink: $\dot{m}_k(t) = \frac{dm_k}{dt}$
$\delta(\mathbf{r})$	Dirac delta function

Alternatively it can be written as:

(5) $$\frac{\partial \rho}{\partial t} + \nabla(\rho \, \mathbf{u}) = \sum_k \rho(t,\mathbf{r}) \cdot \dot{q}_k(t) \cdot \delta(\mathbf{r} - \mathbf{r}_k)$$

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$q_k(t)$	volumetric value of body mass generation rate at k-th source/sink: $q_k(t) = \frac{dV_k}{dt}$

See also

Natural Science / Physics / Mechanics / Continuum mechanics

[Mass Conservation]