# Single-phase pressure diffusion @model

The general form of non-linear single-phase pressure diffusion @model with the finite number of wells is given by:

| | | |
|---|---|---|
| (1) $\phi \cdot c_t \cdot \partial_t p + \nabla \mathbf{u} + c \cdot (\mathbf{u} \, \nabla p) = \sum_k q_k(t) \cdot \delta(\mathbf{r} - \mathbf{r}_k)$ | (2) $\mathbf{u} = - M \cdot (\nabla p - \rho \, \mathbf{g})$ | (3) $\mathrm{F}_\Gamma(p, \mathbf{u}) = 0$ |

where

| | | | |
|---|---|---|---|
| $p(t, \mathbf{r})$ | reservoir pressure | $t$ | time |
| $\rho(\mathbf{r}, p)$ | fluid density | $\mathbf{r}$ | position vector |
| $\phi(\mathbf{r}, p)$ | effective porosity | $\mathbf{r}_k$ | position vector of the $k$-th source |
| $c_t(\mathbf{r}, p)$ | total compressibility | $\delta(\mathbf{r})$ | Dirac delta function |
| $M = k/\mu$ | phase mobility | $\nabla$ | gradient operator |
| $k$ | formation permeability to a given fluid | $\mathbf{g}$ | gravity vector |
| $\mu$ | dynamic viscosity of a given fluid | $\mathbf{u}$ | fluid velocity under Darcy flow |
| $q_k(t)$ | sandface flowrates of the $k$-th well | $\Gamma$ | reservoir boundary |
| $\mathrm{F}_\Gamma(p, \mathbf{u})$ | reservoir boundary flow condition through the reservoir boundary $\Gamma$, which is usually the aquifer or gas cap | | |

Derivation of Single-phase pressure diffusion @model

The alternative form is to write down equations (1) and (2) in reservoir volume outside wellbore and match the solution to the fluid flux through the well-reservoir contact:

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|---|---|
| (4) $\phi \cdot c_t \cdot \partial_t p + \nabla \mathbf{u} + c \cdot (\mathbf{u} \, \nabla p) = 0$ | (5) $\mathbf{u} = - M \cdot (\nabla p - \rho \, \mathbf{g})$ |
| (6) $\int_{\Sigma_k} \mathbf{u} \, d\mathbf{\Sigma} = q_k(t)$ | (7) $\mathrm{F}_\Gamma(p, \mathbf{u}) = 0$ |

where

| | |
|---|---|
| $\Sigma_k$ | well-reservoir contact of the $k$-th well |
| $d\mathbf{\Sigma}$ | normal vector of differential area on the well-reservoir contact, pointing inside wellbore |
| $q_k(t)$ | sandface flowrates at the $k$-th well (could be injecting to or producing from the reservoir ) |

Physical models of pressure diffusion can be split into two categories: Newtonian and Rheological (non-Newtonian) based on the fluid stress model.

Mathematical models of pressure diffusion can be split into three categories: Linear, Pseudo-Linear and Non-linear.

These models are built using Numerical, Analytical or Hybrid pressure diffusion solvers.

Many popular 1DR solutions can be approximated by Radial Flow Pressure Diffusion @model which has a big methodological value.

The simplest analytical solutions for pressure diffusion are given by 1DL Linear-Drive Solution (LDS) and 1DR Line Source Solution (LSS).

## See also

Physics / Mechanics / Continuum mechanics / Fluid Mechanics / Fluid Dynamics / Pressure Diffusion / Pressure Diffusion @model

[ Aquifer Drive Models ] [ Gas Cap Drive Models ]

[ Linear single-phase pressure diffusion @model ]