

Non-linear single-phase pressure diffusion for turbulent flow @model

The general form of non-linear [single-phase pressure diffusion @model](#) is given by:

$$(1) \quad \phi \cdot c_t \cdot \partial_t p - \nabla (M \cdot (\nabla p - \rho \cdot \mathbf{g})) - c \cdot M \cdot (\nabla p)^2 = \sum_k q(\mathbf{r}) \cdot \delta(\mathbf{r} - \mathbf{r}_k)$$

with non-linear dependence of [fluid mobility](#) M on [reservoir pressure](#) p and spatial [pressure](#) gradient ∇p :

$$(2) \quad M = k_{air}(\mathbf{r}) M_r(p, \nabla p)$$

and non-linear dependence of [compressivity](#) β and [compressibility](#) c_t on [reservoir pressure](#) p :

$$(3) \quad \beta = c_t(\mathbf{r}, p) \cdot \phi(\mathbf{r}, p)$$

$$(4) \quad c_t(\mathbf{r}, p) = c_r(\mathbf{r}, p) + \sum_{\alpha} s_{\alpha}(\mathbf{r}) c_{\alpha}(p)$$

where

$M(p, \nabla p)$	Fluid mobility as function of reservoir pressure p and spatial pressure gradient ∇p
$M_r(p, \nabla p)$	Relative mobility as function of reservoir pressure p and spatial pressure gradient ∇p
$\beta(p)$	Compressivity as function of reservoir pressure p
$c_t(\mathbf{r}, p)$	Total compressibility as function of reservoir pressure p and location $\mathbf{r}$
$c_r(\mathbf{r}, p)$	Rock compressibility as function of reservoir pressure p and location $\mathbf{r}$
$c_{\alpha}(p)$	α-phase compressibility as function of reservoir pressure p for $\alpha = \{w, o, g\}$
$s_{\alpha}(\mathbf{r})$	α-phase reservoir saturation for $\alpha = \{w, o, g\}$
$\phi_e(\mathbf{r}, p)$	Effective porosity as function of reservoir pressure p and location $\mathbf{r}$
$k_{air}(\mathbf{r})$	Formation permeability at initial formation pressure p_0 as function of location $\mathbf{r}$
$\mu(p_0)$	Dynamic fluid viscosity at initial formation pressure p_0
$\xi(p, \nabla p)$	Some function of reservoir pressure p and spatial pressure gradient ∇p with the following asymptotic behaviour: $\xi(p \rightarrow p_0, \nabla p \rightarrow 0) \rightarrow 1$

The same account for non-linearity can be applied for [non-linear multi-phase pressure diffusion](#) when [Pressure Diffusion Model Validity Scope](#) is met and multi-phase pressure dynamics can be modeled as effective single-phase pressure dynamics.

Below is the list of popular physical phenomena and their mathematical models which can be covered by (1) model.

Forchheimer Equation

Pressure diffusion equation is going to be:

$$c_t \phi_e \frac{\partial p}{\partial t} = \nabla \left(\frac{k(\nabla p)}{\mu} \nabla p \right)$$

where

$k(\nabla p)$	Dynamic fluid viscosity as function of reservoir pressure p
$k(p)$	Formation permeability as function of reservoir pressure p
$c_f(p)$	Total compressibility as function of reservoir pressure p

See also

[Pressure diffusion](#) / [Pressure Diffusion @model](#) / [Single-phase pressure diffusion model](#) / [Non-linear single-phase pressure diffusion @model](#)