# Non-linear single-phase pressure diffusion for gradient rocks @model

The general form of non-linear single-phase pressure diffusion model is given by:

(1) $$\beta(\mathbf{r}, p) \frac{\partial p}{\partial t} = \nabla \left( M(\mathbf{r}, p, \nabla p) \cdot \nabla p \right)$$

with non-linear dependence of fluid mobility $M$ on reservoir pressure $p$ and spatial pressure gradient $\nabla p$:

(2) $$M = k_{air}(\mathbf{r}) \, M_r(p, \nabla p)$$

and non-linear dependence of compressivity $\beta$ and compressibility $c_t$ on reservoir pressure $p$ :

(3) $$\beta = c_t(\mathbf{r}, p) \cdot \phi(\mathbf{r}, p)$$

(4) $$c_t(\mathbf{r}, p) = c_r(\mathbf{r}, p) + \sum_\alpha s_\alpha(\mathbf{r}) c_\alpha(p)$$

where

| | |
|---|---|
| $M(p, \nabla p)$ | Fluid mobility as function of reservoir pressure $p$ and spatial pressure gradient $\nabla p$ |
| $M_r(p, \nabla p)$ | Relative mobility as function of reservoir pressure $p$ and spatial pressure gradient $\nabla p$ |
| $\beta(p)$ | Compressivity as function of reservoir pressure $p$ |
| $c_t(\mathbf{r}, p)$ | Total compressibility as function of reservoir pressure $p$ and location $\mathbf{r}$ |
| $c_r(\mathbf{r}, p)$ | Rock compressibility as function of reservoir pressure $p$ and location $\mathbf{r}$ |
| $c_\alpha(p)$ | $\alpha$-phase compressibility as function of reservoir pressure $p$ for $\alpha = \{w, o, g\}$ |
| $s_\alpha(\mathbf{r})$ | $\alpha$-phase reservoir saturation for $\alpha = \{w, o, g\}$ |
| $\phi_e(\mathbf{r}, p)$ | Effective porosity as function of reservoir pressure $p$ and location $\mathbf{r}$ |
| $k_{air}(\mathbf{r})$ | Formation permeability at initial formation pressure $p_0$ as function of location $\mathbf{r}$ |
| $\mu(p_0)$ | Dynamic fluid viscosity at initial formation pressure $p_0$ |
| $\xi(p, \|\nabla p\|)$ | Some function of reservoir pressure $p$ and spatial pressure gradient $\nabla p$ with the following asymptotic behaviour: $\xi(p \rightarrow p_0, \|\nabla p\| \rightarrow 0) \rightarrow 1$ |

The same account for non-linearity can be applied for non-linear multi-phase pressure diffusion when Pressure Diffusion Model Validity Scope is met and multi-phase pressure dynamics can be modeled as effective single-phase pressure dynamics.

Below is the list of popular physical phenomena and their mathematical models which can be covered by (1) model.

## Dependance on pressure gradient

Pressure diffusion equation is going to be:

$$\backslash c_t \phi_e \frac{\partial p}{\partial t} = \nabla(\frac{k(\nabla p)}{\mu} \nabla p)$$

where

| | |
|---|---|
| $k(\nabla p)$ | Dynamic fluid viscosity as function of reservoir pressure $p$ |
| $k(p)$ | Formation permeability as function of reservoir pressure $p$ |
| $c_f(p)$ | Total compressibility as function of reservoir pressure $p$ |

## See also

Pressure diffusion / Pressure Diffusion @model / Single-phase pressure diffusion model / Non-linear single-phase pressure diffusion @model