Convolution @math

@wikipedia

A function $f * g$ of real-number argument t, compiled from the two functions $f(t)$ and $g(t)$ by specific integration:

(1) $$(f * g)(t) = \int_{-\infty}^{\infty} f(t-\tau)\, g(\tau)\, d\tau$$

For functions f, g supported on only $[0, \infty)$ (i.e., zero for negative arguments), the integration limits can be truncated, resulting in:

(2) $$(f * g)(t) = \int_{0}^{t} f(t-\tau)\, g(\tau)\, d\tau$$

Properties

(3) $$\int_{0}^{t} f(t-\tau)\, g(\tau)\, d\tau = \int_{0}^{t} f(\tau)\, g(t-\tau)\, d\tau$$

(4) $$\int_{0}^{t} f(t-\tau)\, \dot{h}(\tau) d\tau = f(0)h(t) - f(t)h(0) - \int_{0}^{t} \dot{f}(t-\tau)\, h(\tau)\, d\tau$$

where $\dot{g}()$ means derivative by the whole argument.

Discrete form

For the functions $\{f_n = f(t_n)\}$, $\{g_n = g(t_n)\}$, $n = 1..N$ defined over the discrete time grid $\{t_n\}, n = 1..N$ the convolution equation is taking the discrete form:

(5) $$(f * g)(t_n) = \sum_{m=0}^{n} f(t_n - \tau_m)\, g(\tau_m)\, (t_n - \tau_m)$$

See also

Formal science / Mathematics