Derivation of Single-phase pressure diffusion @model

We start with the reservoir flow continuity equation:

(1)	$\displaystyle \frac{\partial(\rho\phi)}{\partial t} + \nabla(\rho\, \mathbf{u}) = \sum_k \dot m_k(t) \cdot \delta(\mathbf{r} - \mathbf{r}_k)$

percolation model:

(2)	$\mathbf{u} = -M \cdot (\nabla p - \rho\, \mathbf{g})$

and the reservoir boundary flow condition:

(3)	$\mathrm{F}_\Gamma(p, \mathbf{u}) = 0$

where

Σ_k	well-reservoir contact of the k-th well
$d\mathbf{\Sigma}$	normal vector of differential area on the well-reservoir contact, pointing inside wellbore
$\dot m_k(t)$	mass flowrate at k-th well $\dot m_k(t) = \rho(p) \cdot q_k(t)$
$q_k(t)$	sandface flowrate at k-th well
$\rho(p)$	fluid density as function of reservoir fluid pressure p

Then use the following equality:

(4) $$d(\rho\,\phi) = \rho\, d\phi + \phi\, d\rho = \rho\,\phi \left(\frac{d\phi}{\phi} + \frac{d\rho}{\rho} \right) = \rho\,\phi \left(\frac{1}{\phi}\frac{d\phi}{dp}\, dp + \frac{1}{\rho}\frac{d\rho}{dp}\, dp \right) = \rho\,\phi\,(c_\phi\, dp + c\, dp) = \rho\,\phi\, c_t\, dp$$

where

$c_t = \mathrm{c}_\phi + c$	total compressibility

to arrive at:

(5) $\displaystyle \rho\,\phi\, c_t \cdot \frac{\partial p}{\partial t} + \nabla(\rho\, \mathbf{u}) = \sum_k \rho\, q_k(t) \cdot \delta(\mathbf{r} - \mathbf{r}_k)$	(6) $\mathrm{F}_\Gamma(p, \mathbf{u}) = 0$

The left-hand side of equation (5) can be transformed in the following way:

(7) $$\nabla(\rho\, \mathbf{u}) = \rho\, \nabla\, \mathbf{u} + (\nabla \rho,\, \mathbf{u}) = \rho\, \nabla\, \mathbf{u} + \frac{d\rho}{dp} \cdot (\nabla p,\, \mathbf{u}) = \rho\, \nabla\, \mathbf{u} + \rho\, c \cdot (\nabla p,\, \mathbf{u})$$

where $c(p) = \dfrac{1}{\rho}\dfrac{d\rho}{dp}$ is fluid compressibility.

By using the Dirac delta function property: $f(x) \cdot \delta(x - x_0) = f(x_0) \cdot \delta(x - x_0)$ the right-hand side of equation (5) can be transformed in the following way:

(8) $\sum_k \rho(p(t,\mathbf{r})) \cdot q_k(t) \cdot \delta(\mathbf{r}-\mathbf{r}_k) = \sum_k \rho(p(t,\mathbf{r}_k)) \cdot q_k(t) \cdot \delta(\mathbf{r}-\mathbf{r}_k) = \sum_k \rho(p(t,\mathbf{r})) \cdot q_k(t) \cdot \delta(\mathbf{r}-\mathbf{r}_k) = \rho(p) \cdot \sum_k q_k(t) \cdot \delta(\mathbf{r}-\mathbf{r}_k)$

Substituting (7) and (8) into (5) and reducing the density $\rho(p)$ one arrives to:

(9) $\phi\, c_t \cdot \dfrac{\partial p}{\partial t} + \nabla \mathbf{u} + c \cdot (\mathbf{u}\, \nabla p) = \sum_k q_k(t) \cdot \delta(\mathbf{r}-\mathbf{r}_k)$	(10) $\mathrm{F}_\Gamma(p, \mathbf{u}) = 0$

See also

Physics / Mechanics / Continuum mechanics / Fluid Mechanics / Fluid Dynamics / Pressure Diffusion / Pressure Diffusion @model / Single-phase pressure diffusion @model