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2 authors:

J. Dirker
University of Pretoria

Josua P Meyer
University of Pretoria

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Convective Heat Transfer Coefficients in Concentric Annuli

J. DIRKER and J. P. MEYER
Department of Mechanical and Aeronautical Engineering, University of Pretoria, Pretoria, South Africa

The geometric shape of a passage's cross-section has an effect on its convective heat transfer capabilities. For concentric annuli, the diameter ratio of the annular space plays an important role. The purpose of this study was to determine to what extent research has been done on convective heat transfer in smooth concentric annuli and, if possible, to improve on or contribute to existing theories. It was found that although various correlations exist, they are not in good agreement. For this study, experiments were conducted with a wide range of annular diameter ratios. The Wilson plot method was used to develop a convective heat transfer correlation for annular diameter ratios of 1.7 to 3.2. For Reynolds numbers (based on the hydraulic diameter), in the range of 4000 to 30000, the deduced correlation predicted Nusselt numbers accurately within 3% of experimental values.

Since the early 1900s, many researchers have investigated heat transfer in annuli, particularly in order to find correlations that can describe the Nusselt number and convective heat transfer for a wide range of flow conditions and annular diameter ratios. Having a direct correlation could replace the time-consuming process of finding the annular convective heat transfer coefficient by means of the Wilson plot method [1] or other methods. A summary of some of the proposed correlations available is given in Table 1.

Most of the proposed equations for calculating the Nusselt number are functions of the annular diameter ratio, a (defined as the diameter of inner wall of outer tube, D_2, over the diameter of outer wall of inner tube, D_1), the Reynolds number, and the Prandtl number correspond with the Dittus-Boelter type form.

The correlations given in Table 1, all of which are reported to be valid for water, were compared for a case with an annular diameter ratio of 2 and a Prandtl number of 3.37, which is given in Figure 1. All correlations predict an almost linear increase in the Nusselt number with an increase in the Reynolds number. Compared to the other predictions, the equation by Foust and Christian [4] overpredicts the Nusselt number by a factor of approximately three. When the predictions of Foust and Christian [4] are omitted, a difference in predicted values of $\pm 20\%$ relative to the average predicted value is found to exist. The same trend was found to be true for a wide range of annular diameter ratios and Prandtl numbers.

Address correspondence to Prof. J. P. Meyer, the Department of Mechanical and Aeronautical Engineering, University of Pretoria, Pretoria 0002, South Africa. E-mail: jmeyer@up.ac.za

The large discrepancies that exist among the various correlations may be due to different levels of accuracy in experimental data, especially in regard to energy balance errors. Also, many of the correlations were derived for a large range of annular diameter ratios, ranging from 1 up to 6800, and for different flow mediums. Over such vast spectrums of fluid properties and geometric influences, it is difficult to derive generalized correlations without losing accuracy.

No literature was found that indicates the existence of an accurate heat transfer correlation for concentric annuli. Thus the purpose of this investigation was to deduce a correlation with which accurate predictions could be made of average Nusselt numbers at the inner annular wall under turbulent flow conditions of water.

EXPERIMENTAL FACILITY

Eight different concentric tube-in-tube heat exchangers, each with a different annular diameter ratio, were used during the experimental investigation. The tube diameters used are given in Table 2. Each heat exchanger had an effective heat transfer length of about 6 m and was operated in a counterflow arrangement with hot water in the inner tube and cold water in the annulus. The heat exchangers were constructed from hard-drawn refrigeration copper tubing and were operated in a horizontal configuration. The cold water entered all annuli perpendicular to the direction of flow.

The inner tubes were kept in concentric positions by employing sets of radial supporting pins along the length of each heat

Table 1 Equations available from literature describing the Nusselt number in a smooth concentric annulus during forced convection

Author(s)	Correlation		Diameter Ratio Range	Reynolds Number Range	Medium
Davis [2]	$^*\mathrm{Nu}_{D_h} = 0.038a^{0.15}(a-1)^{0.2}\mathrm{Re}_{D_h}^{0.8}\,\mathrm{Pr}^{\frac{1}{3}}\left(\frac{\mu}{\mu_w}\right)_o^{0.14}$	(1)	1.18–6800	Not specified	All mediums
McAdams [3]	$\mathrm{Nu}_{D_h} = 0.03105a^{0.15}(a-1)^{0.2}\mathrm{Re}_{D_h}^{0.8}\,\mathrm{Pr}^{\frac{1}{3}}\left(\frac{\mu}{\mu_w}\right)^{0.14}$	(2)	1.18–6800	Not specified	All mediums
Foust and Christian [4]	$^*\mathrm{Nu}_{D_h} = \frac{0.04a}{(a+1)^{0.2}}\mathrm{Re}_{D_h}^{0.8}\,\mathrm{Pr}^{0.4}$	(3)	1.2–1.84	3000–60000	Water
McAdams [3]	$\mathrm{Nu}_{D_h} = 0.023\mathrm{Re}_{D_h}^{0.8}\,\mathrm{Pr}^{\frac{1}{3}}\left(\frac{\mu}{\mu_w}\right)^{0.14}$	(4)	Not specified	Not specified	Not specified
Monrad and Pelton [5]	$\mathrm{Nu}_{D_h} = 0.023\left[\frac{2\ln a - a^2 + 1}{a - \frac{1}{a} - 2a\ln a}\right]\mathrm{Re}_{D_h}^{0.8}\,\mathrm{Pr}^n$	(5)	1.65, 2.45, 17	12000–220000	Water Air
Wiegand et al. [6]	$\mathrm{Nu}_{D_h} = 0.023a^{0.45}\mathrm{Re}_{D_h}^{0.8}\,\mathrm{Pr}^n\left(\frac{\mu}{\mu_w}\right)^{0.14}$	(6)	1–10	Not specified	Fluids: $\mu_{\mathrm{material}} \leq 2\mu_{\mathrm{water}}$
Kays and Leung [7]	Results listed in tables for various conditions.		1–4	10^4–10^6	Not specified
Petukhov and Roizen [8]	$^*\mathrm{Nu}_{D_h} = \frac{0.06759a^{0.16}}{(a+1)^{0.2}}\zeta\mathrm{Re}_{D_h}^{0.8}$ with $\zeta = 1 + 7.5\left(\frac{a-5}{(a+1)\mathrm{Re}_{D_h}}\right)^{0.6}$ for $a \geq 5$; $\zeta = 1$ for $a \leq 5$	(7)	1–14.3	10^4–$3 \cong 10^5$	Air
Dittus and Boelter [9]	$\mathrm{Nu}_{D_h} = 0.023\mathrm{Re}_{D_h}^{0.8}\,\mathrm{Pr}^n$	(8)	Not specified	Not specified	Not specified
Stein and Begell [10]	$\mathrm{Nu}_{D_h,f} = 0.0200a^{0.5}\mathrm{Re}_{D_h,f}^{0.8}\,\mathrm{Pr}_f^{\frac{1}{3}}$	(9)	1.232 , 1.463, 1.694	30000–390000	Water
Crookston et al. [11]	$\mathrm{Nu}_{D_h} = 0.023a^{\frac{1}{4}}\mathrm{Re}_{D_h}^{\frac{3}{4}}\,\mathrm{Pr}^{\frac{1}{3}}$	(10)	10, 16, 31	17000–100000	Air

*Original equations were rewritten as to have the Reynolds and Nusselt numbers based on the annular hydraulic diameter: $D_h = D_2 - D_1$, with $\mathrm{Nu}_{D_h} = hD_h/k$ and h defined by the Newton law of cooling: $\dot{q}'' = h(T_w - T_m)$.

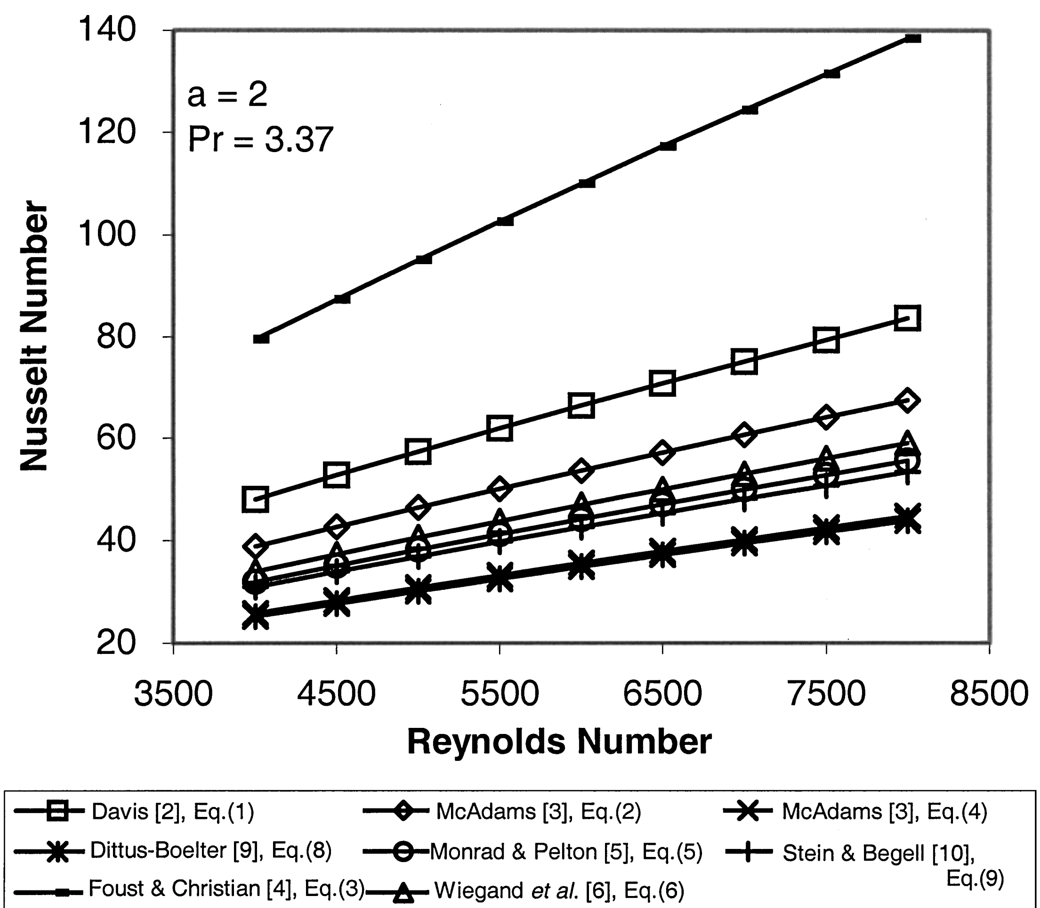

Figure 1 Different predictions of the Nusselt number correlations (see Table 1) as a function of the Reynolds number for $a = 2$ and $\mathrm{Pr} = 3.37$.

Table 2 Tube diameters used for construction of test sections

a [—]	D_1 [mm]	D_2 [mm]	L [m]
1.68	19.05	32.00	6.200
1.76	6.35	11.15	6.170
2.22	6.35	14.10	6.095
2.72	6.35	17.30	6.160
3.20	6.35	20.30	6.170
3.39	9.45	32.00	6.260
4.17	6.35	26.50	6.200
5.04	6.35	32.00	6.240

exchanger at different intervals. By using a symmetrical configuration, possible unbalanced flow patterns were minimized. The size and position of the supporting pins were carefully calculated to minimize possible sagging of the inner tube. The supporting structures occupied at most 6.5% of the cross-sectional area of the smallest annulus. See Figure 2 for a representation of the supporting structure.

Temperature measurements were facilitated by means of K-type thermocouples fixed on the outside surfaces about 30 mm before and after the heat transfer surface of entry and exit regions of the heat exchangers. Three thermocouples evenly spaced around the tube circumference were used at the annular entry and inner tube exit areas, while two were used at the annular exit and inner tube entry regions. Temperature errors were usually less than 0.1 K. Measuring points were sufficiently insulated from the ambient. Hot and cold water had entry temperatures in the vicinity of 50°C and 10°C, respectively, while exit temperatures varied depending on volumetric flow rates.

Volumetric flow rates were measured by using semi-rotary circular piston-type displacement flowmeters with a measuring accuracy of greater than 98%. The positions of the flowmeters were carefully chosen to minimize chaotic flow patterns that could negatively impact measurement accuracy.

Hot water supplied by an onsite hot water storage tank (1000 l), fitted with an electric resistance water heater, was pumped through the inner tube by means of a positive displacement pump and then returned to the storage tank. The hot water flow rates were controlled with a hand-operated bypass system. Similarly, cold water was supplied from a cold water storage tank (1000 l) connected to a chiller and pumped through the annulus by means of two-series connected centrifugal pumps to ensure high flow rates through the test sections before being returned to the storage tank.

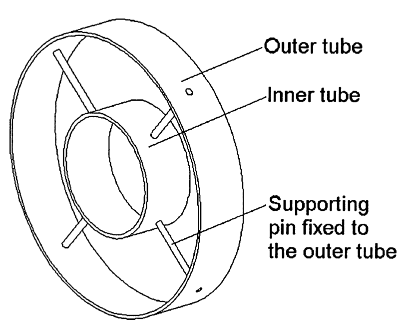

Figure 2 Supporting structure used to maintain concentricity in annuli.

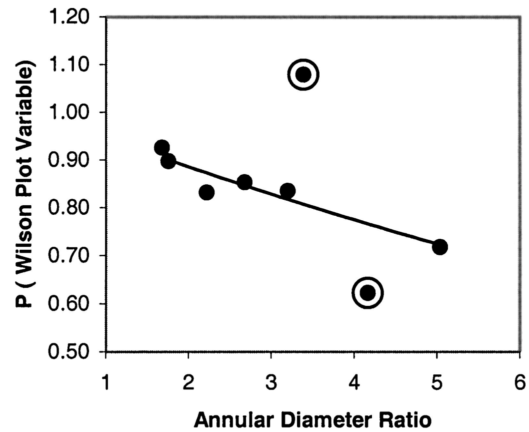

Figure 3 P values obtained from Wilson plot analyses.

EXPERIMENTAL PROCEDURE

Experimental tests [12] were performed at a wide range of flow rate combinations between the inner tube and annulus. As a correlation for the annulus was being deduced, the flow rates in the annulus were of more importance, and a bigger spectrum was covered while ensuring a turbulent flow regime.

Experiments were started with the inner tube flow rate being constant for a number of tests while the annular flow rate was altered through a spectrum. Thereafter, a new inner tube flow rate was used, and the procedure was repeated. After sufficient time was allowed for steady state conditions and good energy balances to be established, the inlet and outlet temperatures of the inner tube and annulus were recorded.

A high level of accuracy in the experimental data was maintained. More than 90% of all data points exhibited an energy balance error of less than 1% between the inner tube and

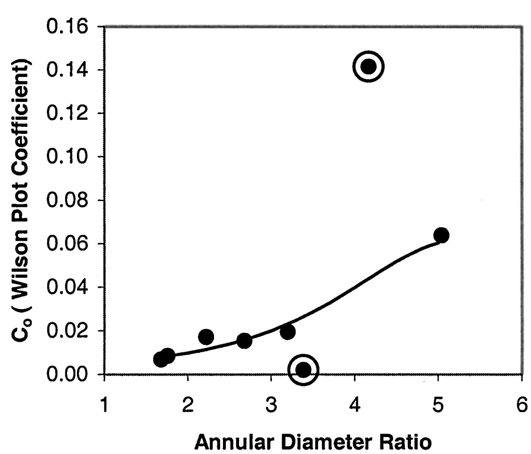

Figure 4 C_o values obtained from Wilson plot analyses.

annular heat transfer rates. A Reynolds number range, based on the hydraulic diameter, of 2600 to 35000 was covered in experiments performed on eight heat exchangers. Suspicious data points were excluded from the analysis process to increase the final accuracy thereof.

PROCESSING OF DATA

The internal and annular Nusselt numbers can be written by means of Eqs. (1) and (2), respectively:

$$\mathrm{Nu}_i = \frac{h_i D_i}{k_i} = C_i \mathrm{Re}_i^{0.8} \mathrm{Pr}_i^{\frac{1}{3}} \left(\frac{\mu}{\mu_w}\right)_i^{0.14} \tag{1}$$

$$\mathrm{Nu}_o = \frac{h_o D_h}{k_o} = C_o \mathrm{Re}_{o,D_h}^{P} \mathrm{Pr}_o^{\frac{1}{3}} \left(\frac{\mu}{\mu_w}\right)_o^{0.14} \tag{2}$$

P, C_i, and C_o are added to account for geometry influences. For the inner tube, the exponent of the Reynolds number was kept at 0.8, as proposed in literature. The modified Wilson plot method developed and described by Briggs and Young [1] was used to process experimental data and to determine the values of particularly C_o and P for each heat exchanger.

More than 95% of all data points were predicted within 3% accuracy by the Wilson plot-obtained correlations given by Eqs. (1) and (2) for the different heat exchangers. All Wilson plot correlations exhibited a median error of less than or in close proximity to 1%. Standard deviances for error values were less than 2%.

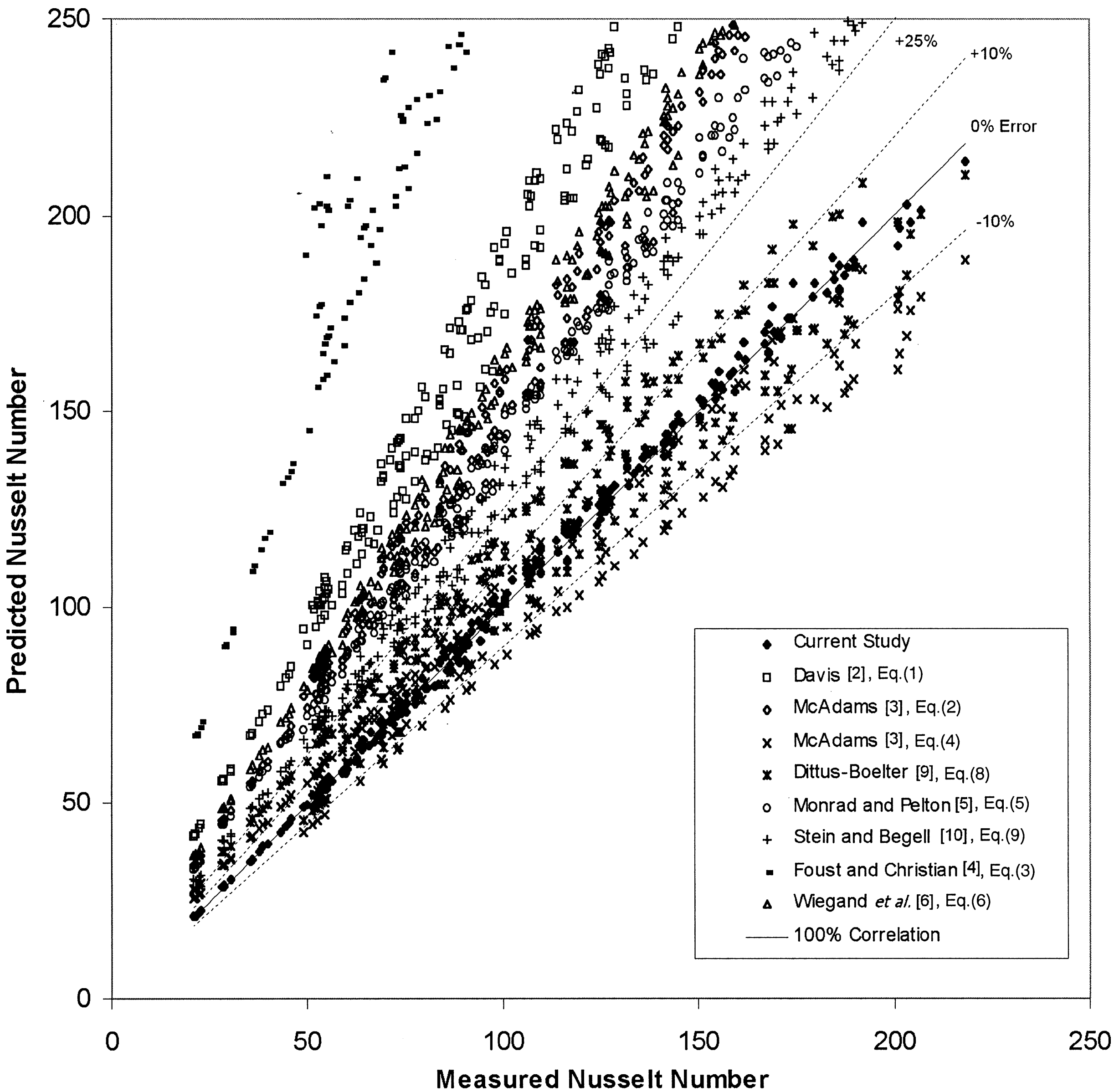

Figure 5 Predictions of correlations cited in literature in terms of measured Nusselt numbers.

DERIVATION OF CORRELATION

As was expected, both P and C_o showed a dependence on the annular diameter ratio. Figures 3 and 4 illustrate the general trends of P and C_o in terms of the diameter ratio. The value of P exhibited a downward trend when the annular diameter ratio was increased. On the other hand, the value of C_o had an upward trend for an increasing annular diameter ratio.

It was observed that results obtained for annular diameter ratios of 4.17 and 3.39 did not agree with the general trend of the rest of the heat exchangers, which are encircled in Figures 3 and 4. These annular cases were rebuilt and the experimental tests repeated. Similar C_o and P values were obtained. A strong possibility exists that concentricity may not have been maintained. This is due to the large distances between the inner and outer tubes for these cases, and the heat exchangers were more prone to internal damage than the heat exchangers with smaller clearances between inner and outer tubes.

From the experimental results though, the behavior of P and C_o for annular diameter ratios below 3.2 can be described relatively precisely with more accuracy and certainty. For ratios greater than 3.2, it is unfortunately not the case, and more experimental investigation is needed.

Using only results for annular ratios below 3.2, it was possible to describe the trend mathematically by evaluating different curve fits. Equations (3) and (4) exhibited the best accuracies and are indicated in Figures 3 and 4 as solid lines.

$$P = 1.013e^{-0.067a} \tag{3}$$

$$C_o = \frac{0.003a^{1.86}}{0.063a^3 - 0.674a^2 + 2.225a - 1.157} \tag{4}$$

By substituting Eqs. (3) and (4) into Eq. (2), a correlation for the prediction of the Nusselt number is produced from where the convective heat transfer coefficient, h_o, can be determined for a particular annular ratio.

The validity of the resulting correlation for the prediction of Nusselt numbers was tested with experimental data from all heat exchangers having an annular diameter ratio of less than 3.2. All predictions were within 3% of experimentally obtained

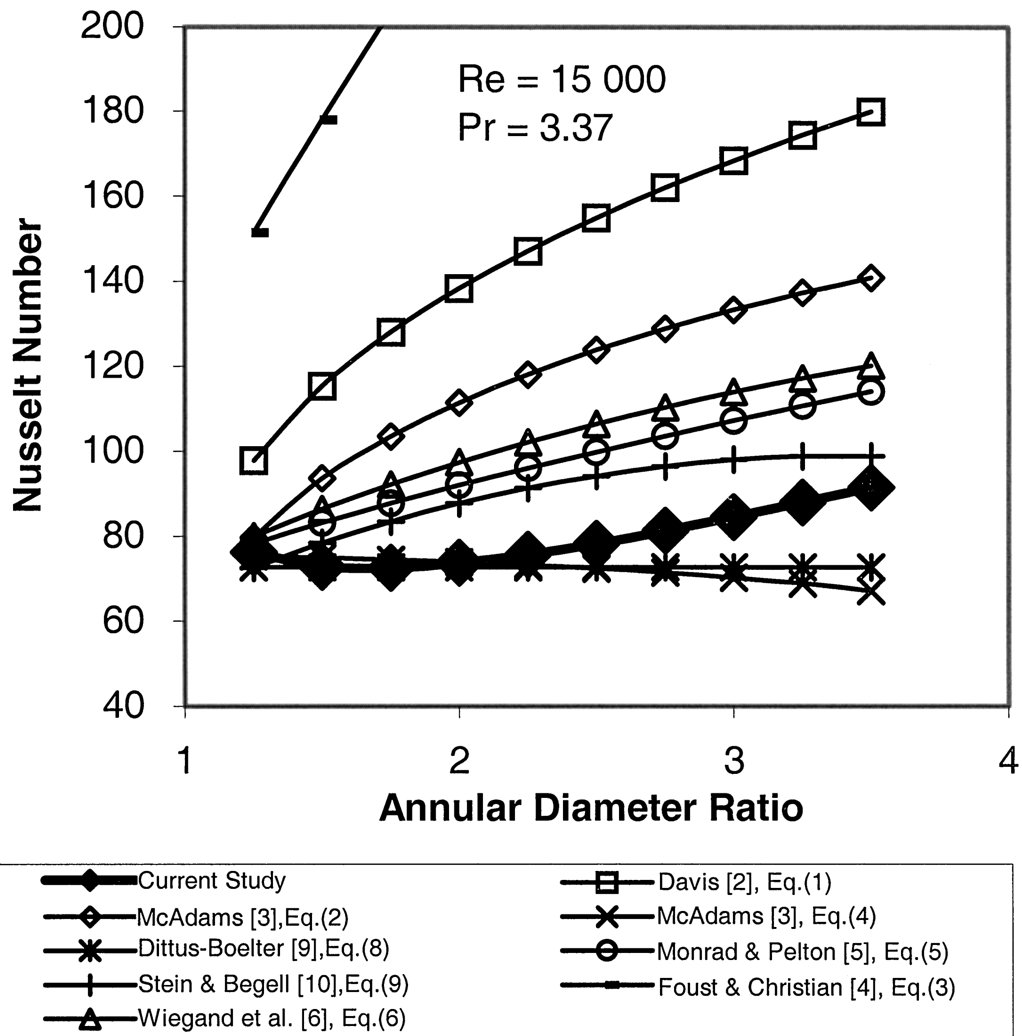

Figure 6 Comparison between the deduced correlation and correlations from literature for a wide range of annular diameter ratios.

values via the Wilson plot analysis procedure. Correlations from the literature (Table 1) were also compared with measured Nusselt numbers in the annular diameter range of 1.7 to 3.2. (See Figure 5.)

Only a small number of the predictions by published correlations were within 10% of the experimentally obtained Nusselt numbers. A large prediction scatter is also exhibited by most of the published correlations. Two main prediction bands are present: the first band in proximity to the measured values consists of predictions by a correlation recommended by McAdams [3] and the Dittus-Boelter correlation [9], and the second band is located outside the 25% range from the measured values and consists of predictions of various authors. Predictions of Foust and Christian [4] had the highest deviance from the measured Nusselt numbers.

The deduced correlation was also compared to correlations in literature for an arbitrary thermal condition over a wide range of annular diameter ratios and Reynolds numbers. For a case where the Reynolds number is 15000 and the Prandtl number is 3.37, the result is shown in Figure 6.

Even though Eq. (4), a correlation recommended by McAdams [3], seems to be independent of a, it contains a viscosity ratio, which in turn is dependent on the wall temperature. In order to maintain the Prandtl number at a constant value, the wall temperature has to adjust to the changing heat transfer area as governed by a. For this reason, the line representing Eq. (4) indicates a slight decrease with an increase in a.

For small annular diameter ratios, up to about 2.5, the predictions correspond well with the correlation by Dittus and Boelter [9] and Eq. (4) by McAdams [3]. In the region of an annular ratio of 3.5, a close agreement exists with the correlation of Stein and Begell [10]. These trends were found to be true for a wide range of Reynolds and Prandtl numbers.

CONCLUSION

As was expected, it was found that the convective heat transfer correlation for an annulus is dependent on the annular diameter ratios. A correlation was deduced from experimental results for the predictions of Nusselt numbers. For diameter ratios between 1.7 and 3.2 and a Reynolds number range of 4000 to 30000, it accurately predicts Nusselt numbers within 3% from the measured values. For small annular diameter ratios below 2.5, the correlation agreed closely with those by Dittus and Boelter [9] and McAdams [3]. For higher annular diameter ratios, it approaches a correlation by Stein and Begell [10].

NOMENCLATURE

a annular diameter ratio, D_2/D_1
C_i inner tube convective heat transfer correlation coefficient (Wilson plot)
C_o annulus convective heat transfer correlation coefficient (Wilson plot)
D_1 diameter of outer wall of inner tube, m
D_2 diameter of inner wall of outer tube, m
D_h hydraulic diameter of annulus $[D_2 - D_1]$, m
D_i inner diameter of inner tube, m
h convective heat transfer coefficient, W/m^{2}K
k thermal conductivity, W/mK
L tube length, m
n exponent of Prandtl number
Nu Nusselt number
P exponent of Reynolds number in Wilson plot function
Pr Prandtl number
$\dot{q}''$ heat flux, W/m^2
Re Reynolds number
T temperature, $^\circ$C or K

Greek Symbols

μ viscosity, Ns/m^2
ζ correlation function

Subscripts

ave average
D_h based on the hydraulic diameter of the annulus
f based on film temperature, $T_f = \frac{1}{2}(T_{ave} + T_w)$
i inner tube side
m average
o annulus side
w wall

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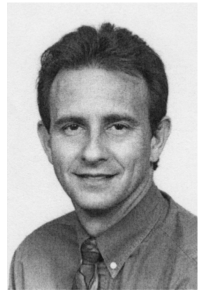
Jaco Dirker obtained his B. Eng. (cum laude) 2000 and M. Eng. (cum laude) 2002 in Mechanical Engineering at the Rand Afrikaans University, South Africa. He is currently working on his Ph.D. in Mechanical Engineering and is doing research involving the cooling of integrated power electronic modules.



Josua Meyer obtained his B. Eng. (cum laude) 1984, M. Eng. (cum laude) 1986, and his Ph.D. (1988) all in Mechanical Engineering from the University of Pretoria, South Africa. He is registered as a Professional Engineer and he completed his military service at the Faculty of Military Science, University of Stellenbosch, where he lectured aerodynamics for pilots in the South African Air Force.

After his military service (1988–1989), he accepted a position as Associate Professor in the Department of Mechanical Engineering at Potchefstroom University in 1990. He was Acting Head and Professor in Mechanical Engineering before accepting a position as professor in the Department of Mechanical and Manufacturing Engineering at the Rand Afrikaans University in 1994. He was the Chairman of Mechanical Engineering from 1999 until June 2002, after which he was appointed as Professor and Head of the Department of Mechanical and Aeronautical Engineering of the University of Pretoria, South Africa.

He specializes in heat transfer, fluid mechanics, and thermodynamic aspects of heating, ventilation, and air conditioning. He is the author and co-author of more than 170 articles, conference papers, and patents, and has received various prestigious awards for his research. He is also a fellow or member of various professional institutes and societies (i.e., South African Institute for Mechanical Engineers, South African Institute for Refrigeration and Air Conditioning, American Society for Mechanical Engineers, and American Society for Air-Conditioning, Refrigeration and Air-Conditioning), and is regularly invited to be a keynote speaker at local and international conferences. He has also received various teaching awards as Lecturer of the Year (at the Potchefstroom University and Rand Afrikaans University). He is a rated researcher by the NRF and an Associate Editor of *Heat Transfer Engineering*.