Free Convection: Overview

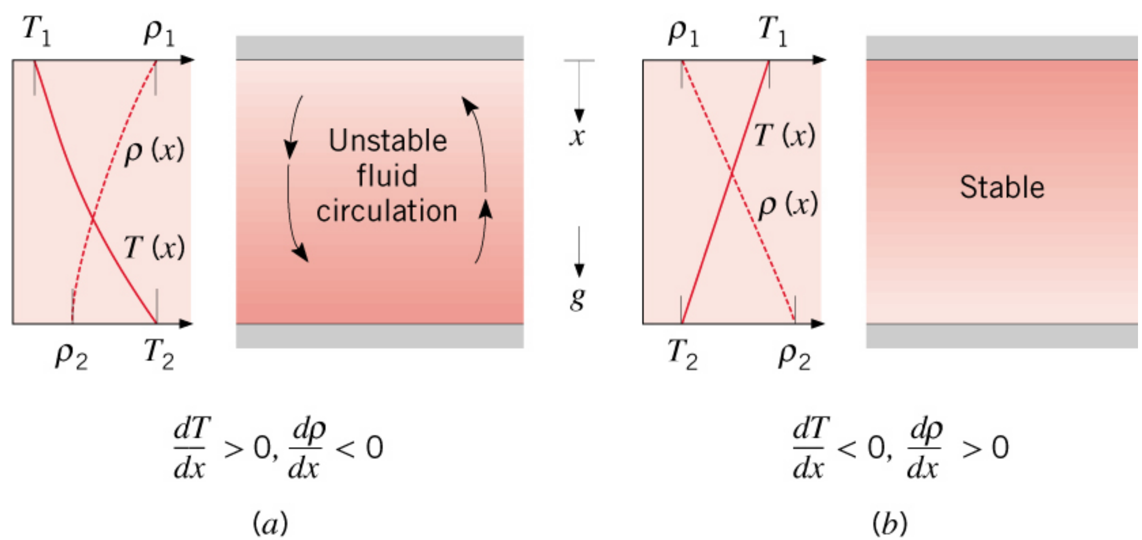
- **Free Convection (or Natural Convection)**
  - ***fluid motion*** induced by ***buoyancy forces***
  - ***buoyancy forces*** arise when there are ***density gradients*** in a fluid and a **body force proportional to density arises**

- **Density Gradient**
  - due to **temperature gradient**

- **Body Force**
  - **gravity** (function of mass)

thermally driven flow

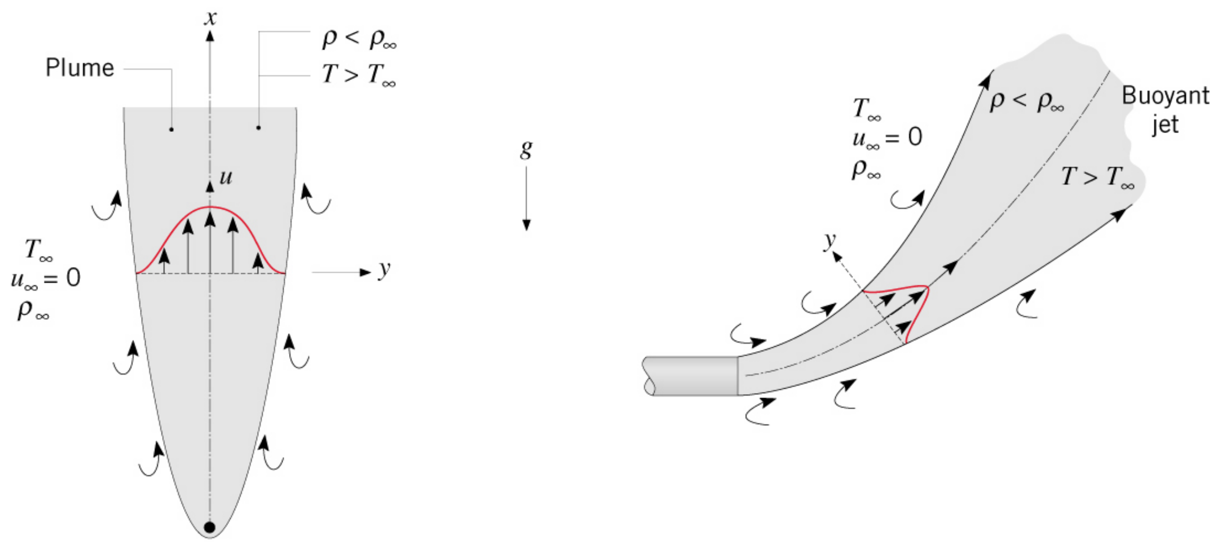
Basic Principle: heavy fluid falls and light fluid rises creating ***vortices***



Free Convection: Overview

- Internal vs. External
  - free convection can be generated in a duct or enclosure (internal)
  - along a free surface (external)
  - in both internal & external cases, free convection can ***interact*** with ***forced convection (mixed convection)***

- Free Boundary Flow
  - occurs in an **extensive quiescent fluid** (*i.e.*, infinite, motionless fluid)
    - no forced convection
  - induces a **boundary layer** on a heated or cooled surface ($T_s \neq T_\infty$)
  - can generate **plumes** and **buoyant jets**



Free Convection: Dimensionless Parameters

Pertinent Dimensionless Parameters

Grashoff Number

$$Gr_L = \frac{g\beta\left(T_s - T_\infty\right)L^3}{\nu^2} \sim \frac{\text{buoyancy force}}{\text{viscous force}}$$

As Grashoff # increases: buoyancy **overcomes** friction and ***induces*** flow

$\nu = \dfrac{\mu}{\rho} \equiv$ kinematic viscosity of fluid

$L \equiv$ characteristic length

$g \equiv$ gravitational constant

$\beta \equiv$ thermal expansion coefficient (fluid property) $\longrightarrow \beta = -\dfrac{1}{\rho}\left(\dfrac{\partial \rho}{\partial T}\right)_p$ [K^{-1}]

Recall Reynolds number

$$\mathrm{Re}_L = \frac{VL}{\nu} \sim \frac{\text{inertial force}}{\text{viscous force}}$$

Gr is analogous to Re

Rayleigh Number

$$Ra_L = Gr_L \mathrm{Pr} = \frac{g\beta\left(T_s - T_\infty\right)L^3}{\nu\alpha}$$

comparable to Peclet number (Pe) for forced convection

Free Convection: Mixed Convection

Mixed Convection

- a condition where both **free** and **forced** convection effects are comparable
- free convection can ***assist***, ***oppose***, or act ***orthogonally*** to forced convection

When is mixed convection **significant**?

- an assessment of **scales** is conducted through a comparison of free and forced **non-dimensional parameters**

$$\frac{\text{buoyancy force}}{\text{inertial force}} \sim \frac{Gr_L}{\mathrm{Re}_L^2}\begin{cases} >>1 & \longrightarrow \text{free convection dominates} \quad Nu_L = f\left(Gr_L, \mathrm{Pr}\right) \\ O(1) & \longrightarrow \text{mixed convection condition} \quad Nu_L = f\left(\mathrm{Re}_L, Gr_L, \mathrm{Pr}\right) \\ <<1 & \longrightarrow \text{forced convection dominates} \quad Nu_L = f\left(\mathrm{Re}_L, \mathrm{Pr}\right) \end{cases}$$

Affect on heat transfer

- forced and free convection **DO NOT** combine linearly

$$Nu_L^n = Nu_{L,forced}^n \pm Nu_{L,free}^n \rightarrow \begin{cases} n = 3 \text{ assisting/opposing} \\ n = 7/2 \text{ or } 4 \text{ transverse} \end{cases}$$

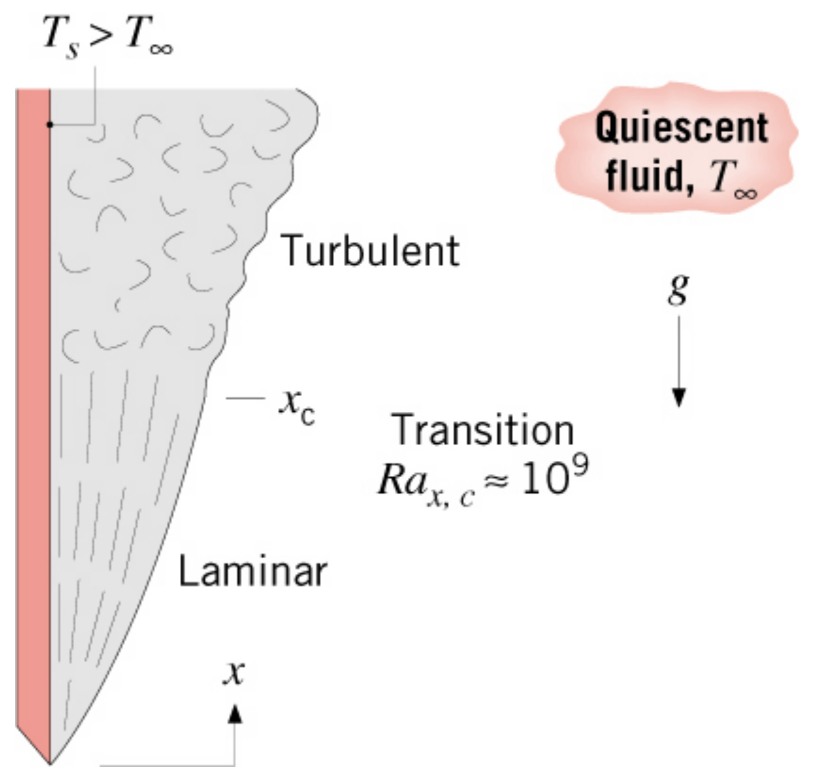
+ ➔ assisting/transverse
- ➔ opposing

Free Convection: Boundary Layers

- **Transition to Turbulence**
  - similar to forced flows, free convection flows can transition from a laminar state to a turbulent state – ***hydrodynamic instabilities***
  - in forced convection, the transition is dependant on **inertial** and **viscous forces**
  - in free convection, amplification of disturbances depends on relative magnitude of **buoyancy** and **viscous forces**

Critical Rayleigh Number

$Ra_{x,c} \approx 10^9$ *vertical flat plate*



Free Convection: External Flow

- Only empirical relations exist for most "real" geometries

$$\overline{Nu}_L = \frac{\bar{h}L}{k} = C Ra_L^n$$

- **Immersed Vertical Plate**
  - ***average*** Nusselt numbers
  - Churchill & Chu correlation: **laminar flow (Ra_L<10^9)**

$$\overline{Nu}_L = 0.68 + \frac{0.670 Ra_L^{1/4}}{\left(1+\left(0.492/\mathrm{Pr}\right)^{9/16}\right)^{4/9}}$$

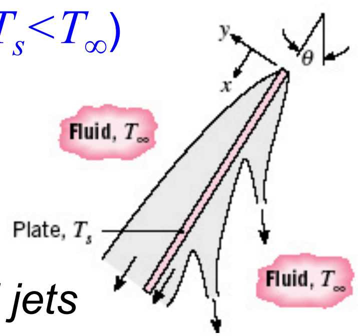
  - Churchill & Chu correlation: **all flow regimes**

$$\overline{Nu}_L = \left[0.825 + \frac{0.387 Ra_L^{1/6}}{\left(1+\left(0.492/\mathrm{Pr}\right)^{9/16}\right)^{8/27}}\right]^2$$

Free Convection: External Flow

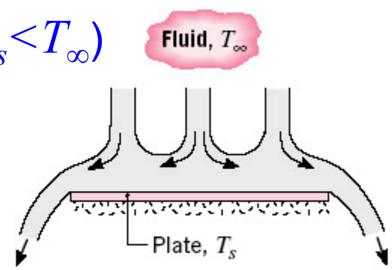
- **Inclined & Horizontal Plates**
  - buoyancy force is **not parallel to plate**
    - horizontal: perpendicular buoyancy force
    - inclined: angled buoyancy force
  - flow and heat transfer depend on
    - thermal condition of plate: **heated** or **cooled**
    - orientation**: facing upward or downward**

cooled ($T_s < T_\infty$)
inclined

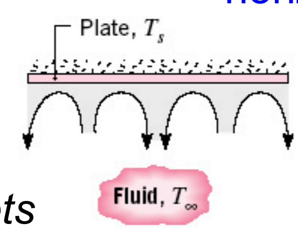


cold fluid jets

cooled ($T_s < T_\infty$)
horizontal
facing up

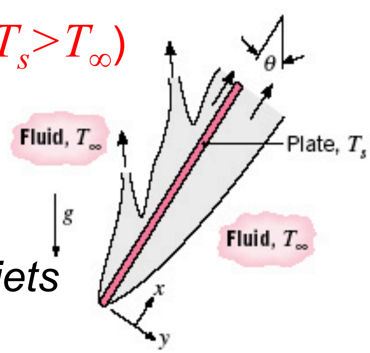


cooled ($T_s < T_\infty$)
horizontal facing
down



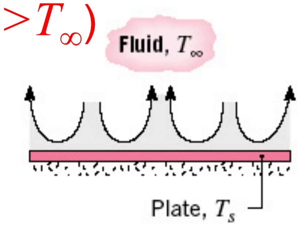
cold fluid jets

heated ($T_s > T_\infty$)
inclined

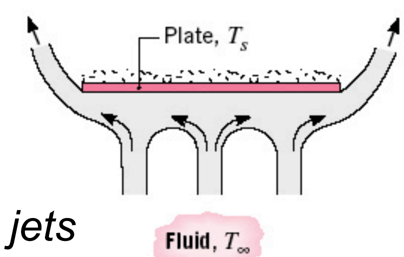


warm fluid jets

heated ($T_s > T_\infty$)
horizontal
facing up



heated ($T_s > T_\infty$)
horizontal facing
down



warm fluid jets

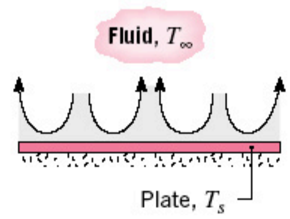
Free Convection: External Flow

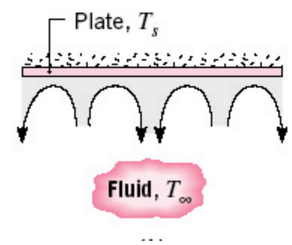
Horizontal Plates

Correlation: **heated/horizontal/facing up** AND **cooled/horizontal/facing down**

$$\overline{Nu}_L = 0.54 Ra_L^{1/4} \rightarrow 10^4 \leq Ra_L \leq 10^7$$

$$\overline{Nu}_L = 0.15 Ra_L^{1/3} \rightarrow 10^7 \leq Ra_L \leq 10^{11}$$

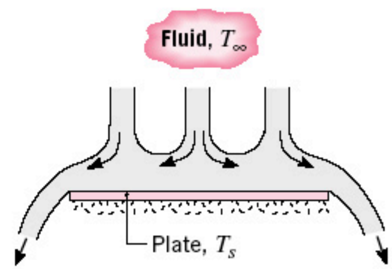


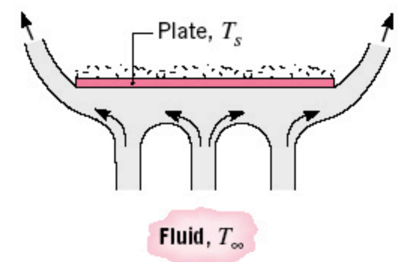


Correlation: **heated/horizontal/facing down** AND **cooled/horizontal/facing up**

$$\overline{Nu}_L = 0.27 Ra_L^{1/4} \rightarrow 10^5 \leq Ra_L \leq 10^{10}$$

note: smaller than above correlation





What is *L* in the Rayleigh number?
Use a characteristic length:

$$L = \frac{A_s}{P}$$

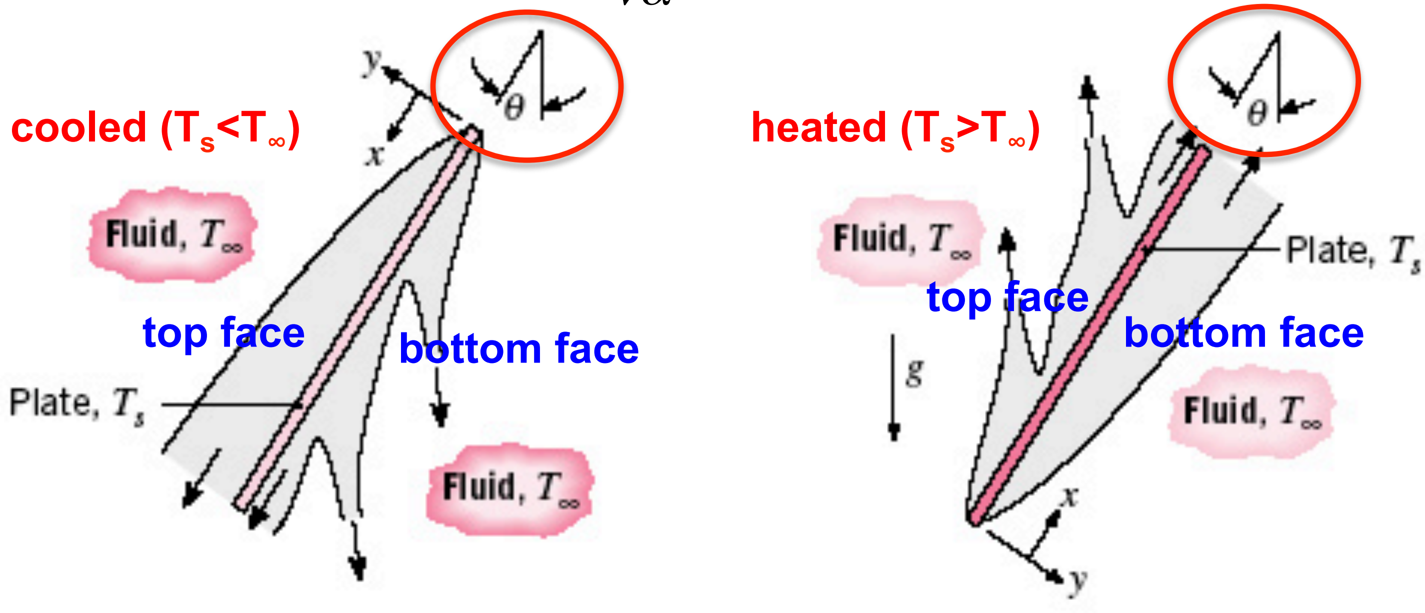
Free Convection: External Flow

Inclined Plates

Correlation: **heated/bottom face only** **AND** **cooled/top face only**

- use immersed vertical plate correlations (Churchill & Chu) but replace ***g*** in the Rayleigh number by ***g*cos*θ***

$$Ra_L = \frac{g\cos\theta\beta\left(T_s - T_\infty\right)L^3}{\nu\alpha} \rightarrow 0 \le \theta \le 60^\circ$$

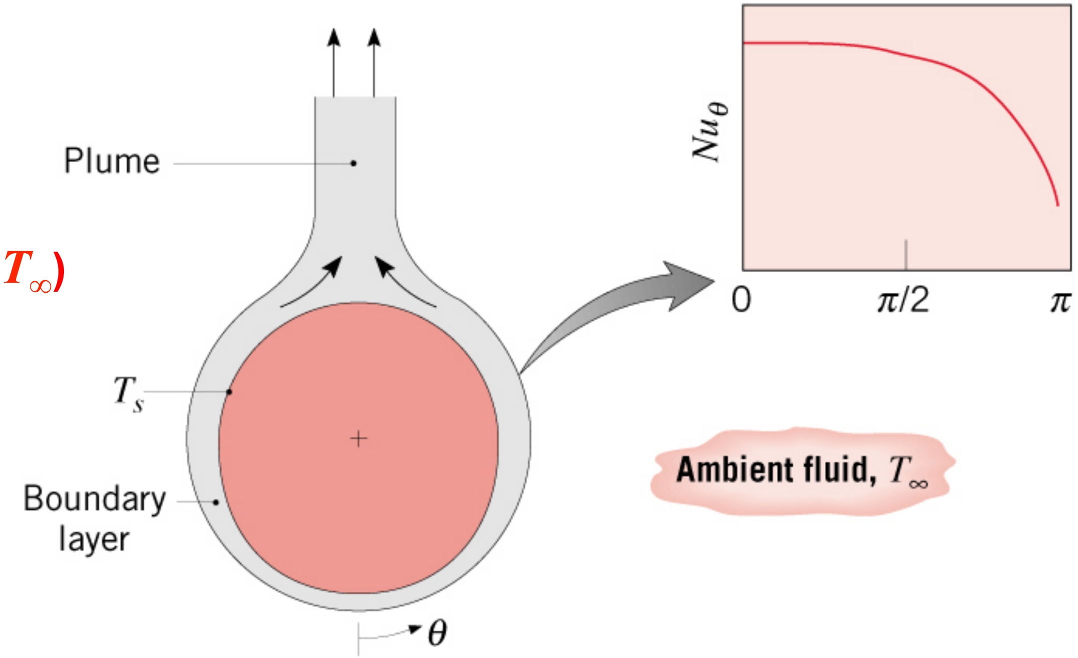


Correlation: **heated/top face only** **AND** **cooled/bottom face only**
flow is **three-dimensional** and **no correlations exist**

Free Convection: External Flow

- **Long Horizontal Cylinder**

heated ($T_s > T_\infty$)



– Rayleigh number based on **diameter of *isothermal cylinder***
– Churchill and Chu correlation: **average Nusselt number**

$$\overline{Nu}_D = \left[0.60 + \frac{0.387 Ra_D^{1/6}}{\left(1 + (0.559/\mathrm{Pr})^{9/16}\right)^{8/27}}\right]^2 \Rightarrow Ra_D < 10^{12}$$

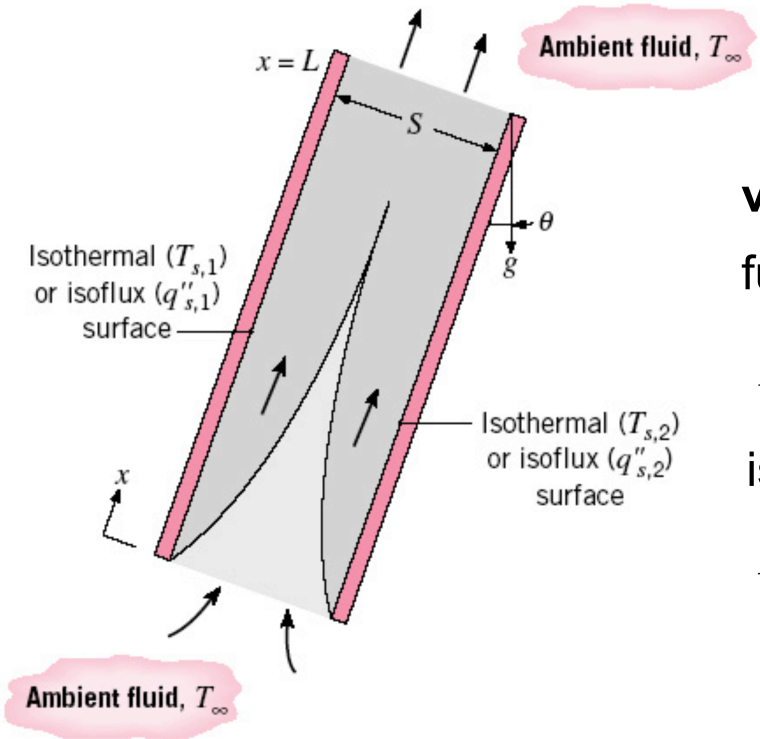
Free Convection: External Flow

- **Sphere**
  - Rayleigh number based on **diameter of *isothermal sphere***
  - Churchill: **average Nusselt number**

$$\overline{Nu}_D = 2 + \frac{0.589 Ra_D^{1/4}}{\left(1+\left(0.469/\mathrm{Pr}\right)^{9/16}\right)^{4/9}} \Rightarrow Ra_D < 10^{11};\ \mathrm{Pr} \geq 0.7$$

Free Convection: Internal Flow

- **Parallel Plates**
  - similar to internal forced convection, buoyancy-induced boundary layers develop on both plates
  - boundary layers can merge to **generate fully developed flow conditions** in sufficiently long (L) and spaced (S) channels ➔ otherwise the channel behaves as two **isolated plates**
  - channels often considered vertical (θ = 0°) or inclined
    - inclined flows more complex and often result in three-dimensional flow



vertical plates

fully developed criteria:

$$Ra_S^* \, S/L \ \text{or} \ Ra_S \, S/L < 10$$

isolated plate criteria:

$$Ra_S^* \, S/L \ \text{or} \ Ra_S \, S/L > 100$$

Free Convection: Internal Flow

- **Parallel Plates – Vertical Channels**
  - both semi-analytical and empirical correlations exist
  - symmetry/asymmetry of boundary conditions play a factor
    - equal or unequal uniform surface temperatures or uniform heat fluxes for both plates

Isothermal

$$\overline{Nu_S} = \left(\frac{q/A}{T_s - T_\infty}\right)\frac{S}{k} = \left[\frac{C_1}{(Ra_S\, S/L)^2} + \frac{C_2}{(Ra_S\, S/L)^{1/2}}\right]^{-1/2}$$

boundary	C_1	C_2
$T_{s,1} = T_{s,2}$	576	2.87
$T_{s,1}, q''_{s,2} = 0$	144	2.87

$$Ra_S = \frac{g\beta(T_1 - T_2)S^3}{\nu\alpha}$$

Isoflux (Uniform Heat Flux)

$$Nu_{S,L} = \left(\frac{q''_s}{T_{s,L} - T_\infty}\right)\frac{S}{k} = \left[\frac{C_1}{Ra_S^*\, S/L} + \frac{C_2}{(Ra_S^*\, S/L)^{2/5}}\right]^{-1/2}$$

boundary	C_1	C_2
$q''_{s,1} = q''_{s,2}$	48	2.51
$q''_{s,1}, q''_{s,2} = 0$	24	2.51

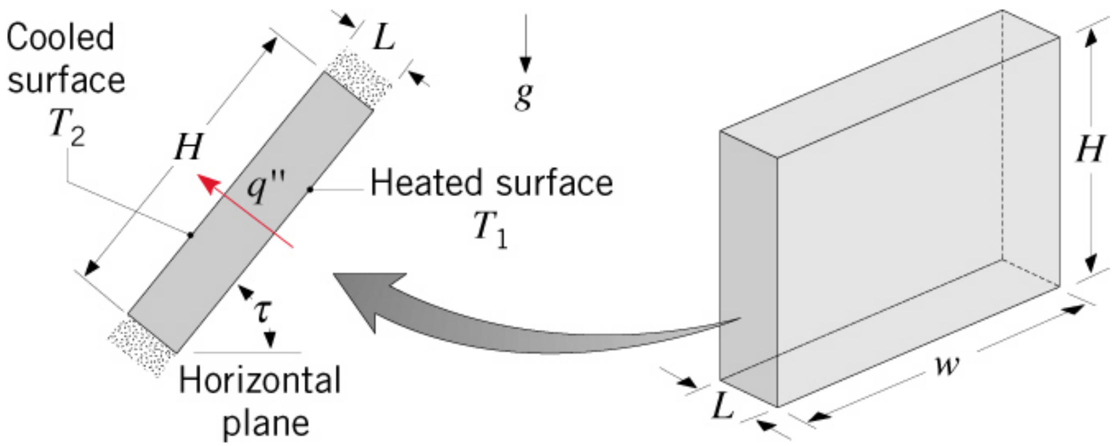
$$Ra_S^* = \frac{g\beta q''_s S^4}{k\nu\alpha}$$

Free Convection: Internal Flow

Only empirical relations exist for most "real" geometries

Enclosures – Rectangular Cavities

- characterized by opposing walls of different temperatures with the the remaining walls insulated
- **fluid motion characterized by induced vortices/rotating flow**



horizontal cavity: τ = 0°,180°
vertical cavity: τ = 90°

Rayleigh number a function of distance between heated walls:

$$Ra_L = \frac{g\beta(T_1 - T_2)L^3}{\nu\alpha} \rightarrow T_1 > T_2$$

Heat transfer **across the cavity**:

$$q'' = h(T_1 - T_2)$$

Free Convection: Internal Flow

Enclosures – Rectangular Cavities (Horizontal)

- **heating from below:** $\tau = 0°$
- **critical Rayleigh number** exists below which **the buoyancy forces cannot overcome the viscous forces**

$$Ra_{L,c} = 1708 \rightarrow \begin{cases} Ra_L < Ra_{L,c} \rightarrow \text{no induced fluid motion} \\ Ra_L > Ra_{L,c} \rightarrow \text{induced fluid motion} \end{cases}$$

$Ra_L < Ra_{L,c}$: **thermally stable –** heat transfer due solely to conduction

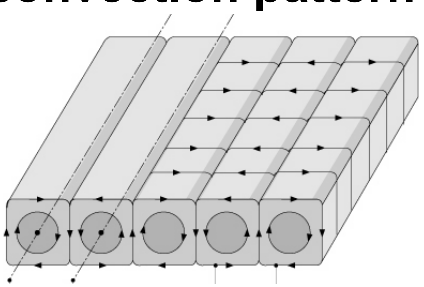
$$\overline{Nu}_L = 1$$

$Ra_{L,c} < Ra_L < 5\times10^4$: **thermally unstable –** heat transfer primarily due to induced fluid motion ➔ **advection**

- thermal instabilities yield **regular convection pattern** in the form of **roll cells**

$$\overline{Nu}_L = 0.069 Ra_L^{1/3} \Pr^{0.074}$$

(first approximation)



$5\times10^4 < Ra_L < 7\times10^9$: **thermally unstable turbulence**

$$\overline{Nu}_L = 0.069 Ra_L^{1/3} \Pr^{0.074}$$

Free Convection: Internal Flow

Enclosures – Rectangular Cavities (Horizontal)

- **Heating from above:** τ = 180°
- **unconditionally thermally stable –** heat transfer due solely to conduction

$$\overline{Nu}_L = 1$$

Enclosures – Rectangular Cavities (Vertical)

- **critical Rayleigh number** exists below which **the buoyancy forces cannot overcome the viscous forces**

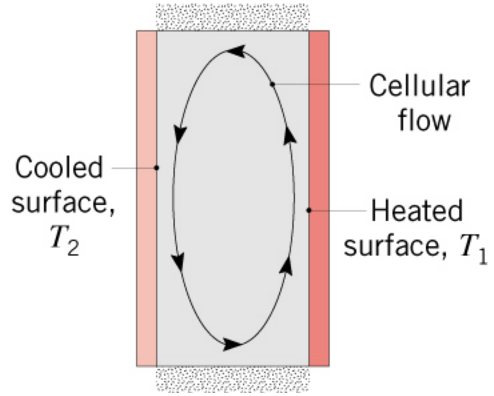
$$Ra_{L,c} = 10^3 \rightarrow \begin{cases} Ra_L < Ra_{L,c} \rightarrow \text{no induced fluid motion} \\ Ra_L > Ra_{L,c} \rightarrow \text{induced fluid motion} \end{cases}$$

$Ra_L < Ra_{L,c}$: **thermally stable –** heat transfer due solely to conduction

$$\overline{Nu}_L = 1$$

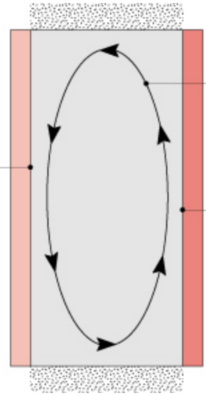
$Ra_{L,c} < Ra_L < 5\times10^4$: **thermally unstable –** heat transfer primarily due to induced fluid motion ➔ **advection**

- a primary flow cell is induced – secondary cells possible at large Rayleigh numbers



Free Convection: Internal Flow

- **Enclosures – Rectangular Cavities (Vertical)**
  - $Ra_{L,c} < Ra_L$: **thermally unstable**
    - Nusselt number a function of cavity geometry and fluid properties



$$\overline{Nu}_L = 0.22\left(\frac{\Pr Ra_L}{0.2+\Pr}\right)^{0.28}\left(\frac{H}{L}\right)^{-1/4} \Rightarrow \begin{bmatrix} 2 < H/L < 10 \\ \Pr < 10^5 \\ 10^3 < Ra_L < 10^{10} \end{bmatrix}$$

$$\overline{Nu}_L = 0.18\left(\frac{\Pr Ra_L}{0.2+\Pr}\right)^{0.29} \Rightarrow \begin{bmatrix} 1 < H/L < 2 \\ 10^{-3} < \Pr < 10^5 \\ 10^3 < Ra_L \Pr/(0.2+\Pr) \end{bmatrix}$$

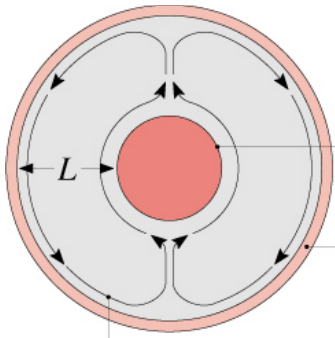
$$\overline{Nu}_L = 0.42 Ra_L^{1/4} \Pr^{0.012}\left(\frac{H}{L}\right)^{-0.3} \Rightarrow \begin{bmatrix} 10 < H/L < 40 \\ 1 < \Pr < 2\times 10^4 \\ 10^4 < Ra_L < 10^7 \end{bmatrix}$$

$$\overline{Nu}_L = 0.046 Ra_L^{1/3} \Rightarrow \begin{bmatrix} 1 < H/L < 40 \\ 1 < \Pr < 20 \\ 10^6 < Ra_L < 10^9 \end{bmatrix}$$

Free Convection: Internal Flow

- **Enclosures – Annular Cavities**
  - **Concentric Cylinders**
    - heat transfer across cavity

$$q' = \frac{2\pi k_{eff}}{\ln(D_o/D_i)}(T_i - T_o)$$



- k_{eff} is **effective thermal conductivity** ➔ *thermal conductivity a stationary fluid would need to transfer the* ***same amount of heat*** *as a moving fluid*

$$Ra_c^* < 100 \Rightarrow \frac{k_{eff}}{k} = 1$$

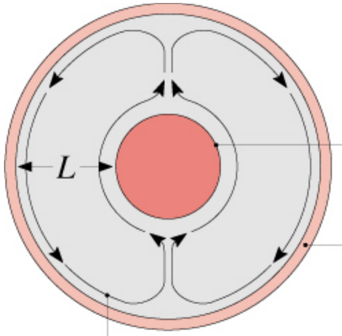
$$100 < Ra_c^* < 10^7 \Rightarrow \frac{k_{eff}}{k} = 0.386\left(\frac{\Pr}{0.861+\Pr}\right)^{1/4}\left(Ra_c^*\right)^{1/4}$$

- ***Critical Rayleigh number:*** $Ra_c^* = \frac{\left[\ln(D_o/D_i)\right]^4}{L^3\left(D_i^{-3/5} + D_o^{-3/5}\right)^5} Ra_L \rightarrow L \equiv (D_o - D_i)/2$

Free Convection: Internal Flow

- **Enclosures – Annular Cavities**
  - **Concentric Spheres**
    - heat transfer across cavity

$$q = \frac{\pi k_{eff} D_i D_o}{L}\left(T_i - T_o\right)$$



- k_{eff} is **effective thermal conductivity** ➔ *thermal conductivity a stationary fluid should have to transfer the* **same amount of heat** *as a moving fluid*

$$Ra_s^* < 100 \Rightarrow \frac{k_{eff}}{k} = 1$$

$$100 < Ra_s^* < 10^4 \Rightarrow \frac{k_{eff}}{k} = 0.74\left(\frac{\Pr}{0.861+\Pr}\right)^{1/4}\left(Ra_s^*\right)^{1/4}$$

- ***Critical Rayleigh number:*** $Ra_s^* = \frac{LRa_L}{\left(D_o/D_i\right)^4\left(D_i^{-7/5} + D_o^{-7/5}\right)^5} \rightarrow L \equiv \left(D_o - D_i\right)/2$