Wellbore Heat Transmission

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ABSTRACT

As fluids move through a wellbore, there is transfer of heat between fluids and the earth due to the difference between fluid and geothermal temperatures. This type of heat transmission is involved in drilling and in all producing operations. In certain cases, quantitative knowledge of wellbore heat transmission is very important.

This paper presents an approximate solution to the wellbore heat-transmission problem involved in injection of hot or cold fluids. The solution permits estimation of the temperature of fluids, tubing and casing as a function of depth and time. The result is expressed in simple algebraic form suitable for slide-rule calculation. The solution assumes that heat transfer in the wellbore is steady-state, while heat transfer to the earth will be unsteady radial conduction. Allowance is made for heat resistances in the wellbore. The method used may be applied to derivation of other heat problems such as flow through multiple strings in a wellbore.

Comparisons of computed and field results are presented to establish the usefulness of the solution.

INTRODUCTION

During the past few years, considerable interest has been generated in hot-fluid-injection oil-recovery methods. These methods depend upon application of heat to a reservoir by means of a heat-transfer medium heated at the surface. Clearly, heat losses between the surface and the injection interval could be extremely important to this process. Not quite so obvious is the fact that every injection and production operation is accompanied by transmission of heat between wellbore fluids and the earth.

Previously, the interpretation of temperature logs[1,2] has been the main purpose of wellbore heat studies. The only papers dealing specifically with long-time injection operations are those of Moss and White[3] and Lesem, *et al.*[4] The purpose of the present study is to investigate wellbore heat transmission to provide engineering methods useful in both production and injection operations, and basic techniques useful in all wellbore heat-transmission problems. The approach is similar to that of Moss and White.[3]

DEVELOPMENT

The transient heat-transmission problem under consideration is as follows. Let us consider the injection of a fluid down the tubing in a well which is cased to the top of the injection interval. Assuming fluid is injected at known rates and surface temperatures, determine the temperature of the injected fluid as a function of depth and time. Consideration of the heat transferred from the injected fluid to the formation leads to the following equations. For liquid,

$$T_1(z,t) = aZ + b - aA + (T_0 + aA - b)e^{-Z/A}; \quad \ldots \quad (1)$$

and for gas,

$$T_1(z,t) = aZ + b - A\left(a + \frac{1}{778c}\right) + \left[T_0 - b + A\left(a + \frac{1}{778c}\right)\right]e^{-Z/A} \quad \ldots \quad (1A)$$

where

$$A = \frac{Wc[k + r_1 U f(t)]}{2\pi r_1 U k} \quad \ldots \quad (2)$$

Eqs. 1, 1A and 2 are developed in the Appendix. These equations were developed under the assumption that physical and thermal properties of the earth and wellbore fluids do not vary with temperature, that heat will transfer radially in the earth and that heat transmission in the wellbore is rapid compared to heat flow in the formation and, thus, can be represented by steady-state solutions.

Special cases of this development have been presented by Nowak[1] and Moss and White.[3] Both references are recommended for excellent background material. Nowak[1] presents very useful information concerning the effect of a shut-in period on subsequent temperatures.

Since one purpose of this paper is to present methods which may be used to derive approximate solutions for heat-transmission problems associated to those specifically considered here, a brief discussion of associated heat problems is also presented in the Appendix. Analysis of the derivation presented in the Appendix will indicate that many terms can be re-defined to modify the solution for application to other problems.

Before Eqs. 1, 1A and 2 can be used, it is necessary to consider the significance of the over-all heat-transfer coefficient U and the time function $f(t)$.

Thorough discussions of the concept of the over-all heat-transfer coefficient may be found in many references on heat transmission. See McAdams[5] or Jakob,[6] for example. Briefly, the over-all coefficient U considers the net resistance to heat flow offered by fluid inside the tubing, the tubing wall, fluids or solids in the annulus, and the casing wall. The effect of radiant heat transfer from the tubing to the casing and resistance to heat flow caused by scale or wax on the tubing or casing may also be included in the over-all coefficient. According to McAdams, on page 136 of Ref. 5,

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[1]References given at end of paper.

SPE 96

$$\frac{1}{U} = \frac{dA_1}{h_1 dA_1} + \frac{x_t dA_1}{k_t dA_t} + \frac{dA_1}{h_2 dA_1'} + \frac{dA_1}{h_2 dA_2} + \frac{x_c dA_1}{k_c dA_c} \quad . . . . (3)$$

The differential areas presented in Eq. 3 are perpendicular to heat flow and, thus, proportional to either radius or diameter measurements. The logarithmic mean area may be determined from

$$A_t = \frac{A_1' - A_1}{\ln (A_1'/A_1)} \quad . . . . (4)$$

where "ln" denotes the natural logarithm.

If the annulus is filled with an insulating material, the third and fourth terms in Eq. 3 should be dropped and a term similar to those for the tubing or casing wall added. Eq. 3 then becomes

$$\frac{1}{U} = \frac{dA_1}{h_1 dA_1} + \frac{x_t dA_1}{k_t dA_t} + \frac{x_a dA_1}{k_a dA_a} + \frac{x_c dA_1}{k_c dA_c} \quad . . . (5)$$

The local heat-transfer coefficients appearing in Eq. 3 (h_1, h_2) may be found from heat-transfer correlations for the particular type of flow, i.e., turbulent, streamline, or free convection. (See pages 168, 190 and 248 of Ref. 5.) If the annulus is under vacuum, the local heat-transfer coefficient for the annulus will be negligible, but heat may be transferred from tubing to casing by radiation. An equivalent local heat-transfer coefficient for the radiation effect may be found on page 63 of Ref. 5. Radiation may be important whether the annulus is under vacuum or filled with gas. If so, the local heat-transfer coefficient for the annulus should be increased by the radiation contribution. It is also possible that any or all of the surfaces of the tubing and casing will be covered by scale and wax. This effect can be included in Eq. 3 by addition of terms similar to those for transfer through the fluid films. The corresponding area term will be the area of the surface covered by the scale or wax. Values for scale or wax coefficients are also presented by McAdams,[5] on page 137.

In many cases, the annulus between the casing and hole is cemented. Because the conductivity of cement may be lower than that of the surrounding earth, a term similar to that for the resistance of pipe or casing wall should appear in the over-all heat-transfer coefficient, Eq. 3. The thickness of cement-filled annulus should be used with the logarithmic mean area of the cement. In this instance, the temperature T_2 will refer to the temperature of the outside surface of the cement and a corresponding radius should be used to evaluate $f(t)$. The conductivity of cement may be estimated from data presented by Jakob, on page 94 of Ref. 6.

For those readers not familiar with the over-all heat-transfer-coefficient concept, the following "rules of thumb" are offered for convenience.

1. The thermal resistance of pipe or casing can often be neglected since the thermal conductivity of steel is much higher than that of other materials in the wellbore or the earth.

2. The thermal resistance of liquid water or condensing steam can often be neglected since heat-transfer film coefficients are so high as to offer little resistance to heat flow (range from about 200 to 2,000 Btu/hr-sq ft-°F).

3. Gas film coefficients and thermal resistance of insulating materials in the wellbore often exert the greatest effect on the over-all coefficient. Gas film coefficients for turbulent flow are often about 2 to 5 Btu/hr-sq ft-°F.

Evaluation of the over-all heat-transfer coefficient is the most difficult step involved in wellbore heat-transmission problems. But certain problems—for example, injection of a liquid down casing—thermal resistance in the wellbore is negligible. In this case, the over-all heat-transfer coefficient can be assumed infinite, and Eq. 2 reduces to

$$A = \frac{Wcf(t)}{2\pi k} \quad . . . . (2A)$$

The problem then becomes simply to find the proper time function $f(t)$. This case is that treated by Moss and White.[3]

The time function $f(t)$ introduced in Eq. 2 may be estimated from solutions for radial heat conduction from an infinitely long cylinder. Such solutions are presented in many texts on heat transmission and are analogous to transient fluid-flow solutions used in reservoir engineering. (See Carslaw and Jaeger,[7] page 283.) Fig. 1 presents $f(t)$ for a cylinder losing heat at constant temperature, a constant heat-flux line source and a cylinder losing heat under the "radiation"[7] or convection boundary condition. As can be seen from Fig. 1 (as well as long-time solutions presented by Carslaw and Jaeger[7]), all three solutions eventually converge to the same line. The convergence time is on the order of one week for many reservoir problems. Thus, the line source solution will often provide a useful result if times are greater than one week. The equation for $f(t)$ for the line source for long times is

$$f(t) = -\ln \frac{r_2'}{2\sqrt{\alpha t}} - 0.290 + 0(r_2'^2/4\alpha t) \quad . . . (5)$$

For estimation of temperatures at times before the convergence time shown on Fig. 1, $f(t)$ should be read from the "radiation"-boundary-condition case at the proper value of (r_1U/k). See the Appendix.

DISCUSSION

The preceding offers an approximate solution to the wellbore heat problem involved in injection of a hot fluid down tubing. Two assumptions appear to be of primary importance: (1) heat flows radially away from the wellbore; and (2) heat flow through various thermal resistances in the immediate vicinity of the wellbore is rapid compared to heat flow in the formation, and can be represented by steady-state solutions. Other assumptions, such as constant thermal and physical properties, appear reasonable.

To test the usefulness of the approximate solution, computed results have been compared with field data.

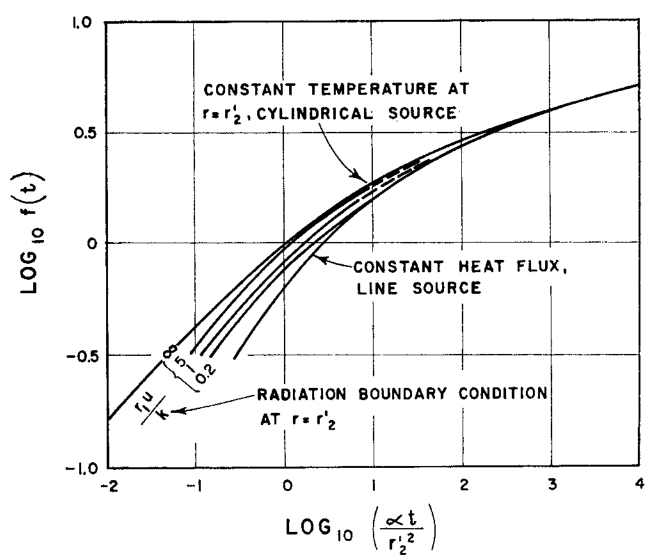

Fig. 1—Transient Heat Conduction in an Infinite Radial System.[7]

COMPARISON OF FIELD TEMPERATURES WITH COMPUTED TEMPERATURES

Following are analyses of field data from a variety of water and gas injections.

Cold Water Injection

Fig. 2 presents a comparison of temperatures measured in a water-injection well with temperatures computed for existing conditions. The water-injection rate at the time of the survey was 4,790 barrels per day; the well had been on injection for a period of approximately 75 days. Water-injection temperature was 58.5°F. As shown on Fig. 2, the computed temperatures were within 1.5°F of the measured temperatures. The reported accuracy of the temperature log was ±2°F. A sample calculation for this case is presented in the Appendix.

Fig. 2 illustrates a point worth concern in certain waterflooding operations. The water entering the interval at approximately 6,500 ft is 55°F cooler than the formation temperature. An approximate calculation of the rate of the water-front advance and the cold-front advance indicates the cold front would move about half the velocity of the water front for many California water floods. Thus, recovery of residual oil behind the water front by continued flooding could be seriously affected by an increase in oil viscosity at the temperature of the cold injected water. (Formation temperature was observed to drop 50°F several hundred feet away from an injection well in a Wilmington water flood.)

Air Injection

Fig. 3 presents a comparison of measured and computed temperatures for an air-injection well. At the time of the survey, air was being injected at 230 Mcf/D and had been injected for a period of six days. Injection temperature was 94°F. As shown on Fig. 3, computed temperatures closely agreed with measured temperatures near the top of the well, but were 8°F higher than measured temperatures at 1,500 ft. The estimated accuracy of temperature measurements was ±5°F. The increase in temperature opposite the injection interval was caused by spontaneous reaction between air and oil which eventually resulted in ignition of the oil.

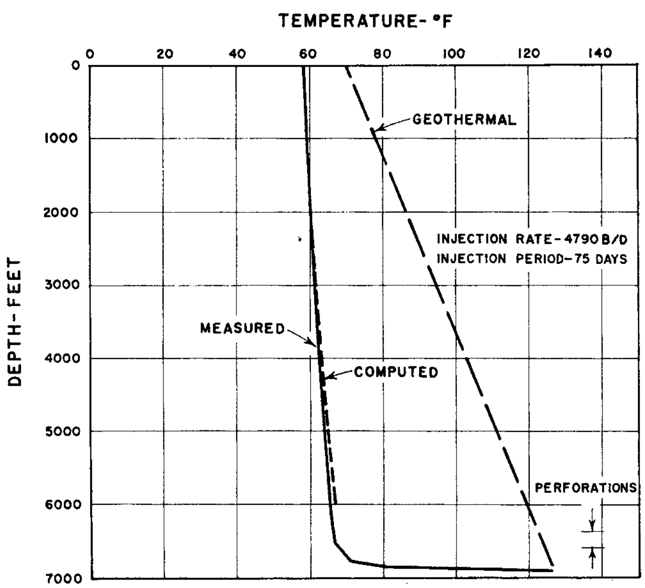

Fig. 2—Measured and Computed Temperatures for a Water-Injection Well.

This case is a particularly interesting one. Air was injected in the casing annulus, and temperatures were measured in the tubing which was plugged on bottom. In addition, sufficient information was available to permit estimation of the effect of mud and cement in the annulus between the hole and casing and the effect of surface pipe on heat transmission.

Hot Natural-Gas Injection

Fig 4 presents a comparison of measured and computed temperatures for injection of hot natural-gas down insulated tubing. This gas-injection project provided the most complete information available for testing the approximate solution. During the year and a half this test was

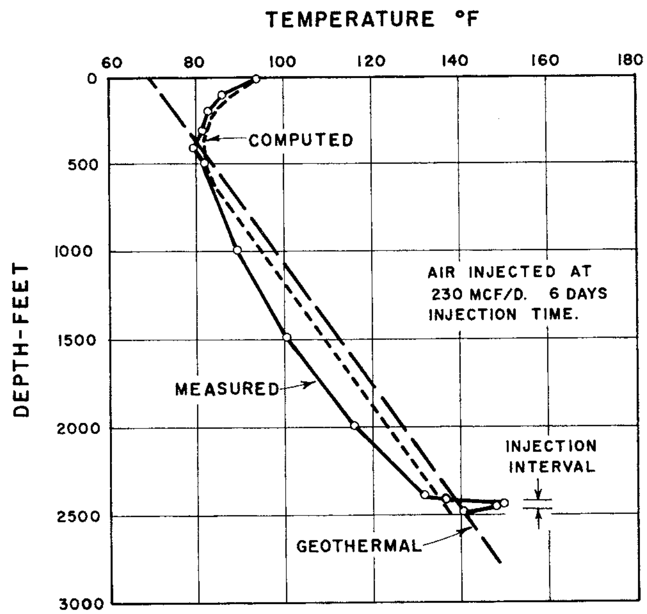

Fig. 3—Measured and Computed Temperatures in an Air-Injection Well (Air Injected at 230 Mcf/D, Six Days Injection Time).

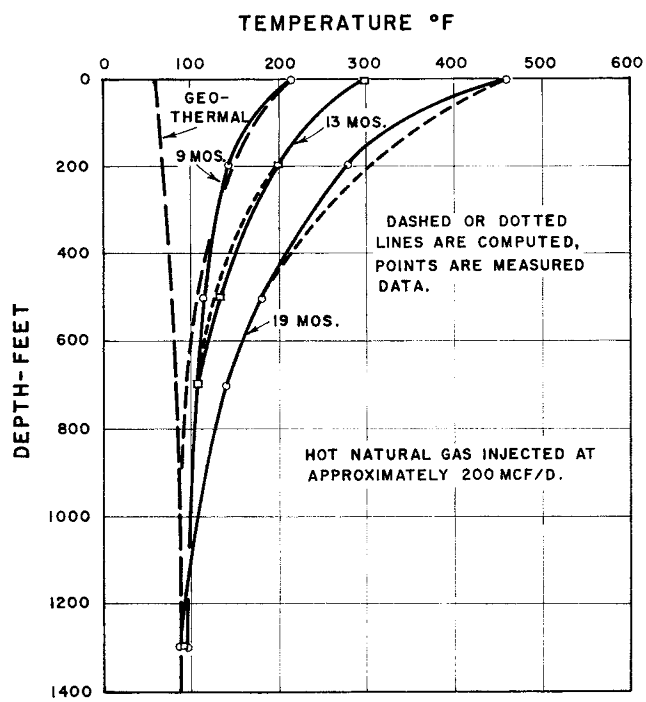

Fig. 4—Measured and Computed Temperatures for Hot Natural-Gas Injection Down Insulated Tubing (Hot Natural Gas Injected at Approximately 200 Mcf/D).

operated, the temperature of the injected gas was increased to almost 500°F, and the gas-injection rate varied from 10 to 215 Mcf/D. Gas was injected down 3-in. tubing. The annulus between the tubing and the 7-in. casing was filled with Perlite.

Measured and computed temperatures are shown on Fig. 4 for three times after start of injection—9, 13 and 19 months. The computed curves are quite similar to the measured curves. Both computed and measured temperatures are below 100°F at 1,300-ft depth throughout the test— despite the surface injection temperature of 460°F. This case illustrates the importance of wellbore heat loss during hot, noncondensable gas injection.

Other sets of field temperatures have been compared with computed temperatures, with results similar to those presented. The three cases presented were selected as representative of the widest conditions tested to date. In view of the reasonable agreement between measured and computed temperatures, it appears that the approximate solution offers a useful method for estimation of temperatures —at least over the ranges of field conditions tested. Further checks of field temperatures and computed temperatures should help define the usefulness of this solution.

HOT FLUID INJECTION

An interesting application of the wellbore heat-transmission problem is estimation of heat losses from the wellbore during injection of a hot fluid for recovery of oil. In addition to wellbore heat loss, vertical heat losses from the producing formation are also important. Although not treated in this paper, several authors have considered vertical heat losses from the formation.[8, 9] Several field pilot tests of steam or hot water injection have been completed, or are in progress.[10, 11]

Of the various heat-transport mediums available, steam or high-pressure hot water appear most attractive. Both steam and hot water have much higher specific-heat capacity than inert gases. However, several questions arise. Will wellbore heat losses be as severe as indicated by Fig. 4? Is it possible to reduce wellbore heat loss to a practical level?

To explore these questions, three cases of steam or hot-water injection have been considered. Before proceeding with these sample cases, it is informative to consider phase relationships for water. Fig. 5 presents a pressure-temperature phase diagram for water in the liquid-vapor region.[12]

Assuming that it is necessary to raise formation temperature to 400°F to achieve satisfactory removal of oil, Fig. 5 indicates that the condition of the water injected will depend upon the injection pressure required. If injection pressure is less than 250 psia, it would be possible to inject steam and take benefit of the high latent-heat content. If injection pressure is above 250 psia, it will be necessary to inject liquid water.

Let us first consider the injection of 500 barrels per day of water at a temperature of 397°F down the casing of a well completed with 7 in., 23-lb casing. If injection pressure is assumed to be 1,000 psi, Fig. 5 indicates that the water will be in the liquid phase. Thus, the previous solution given by Eq. 1 may be applied directly to estimate the temperatures in the well at any time after injection and for any depth. Fig. 6 presents computed temperatures for one week of injection. As shown on Fig. 6, temperatures would decrease severely with depth, indicating a serious heat loss from the hot water.

If 500 BWPD at 397°F are injected at a pressure of 223 psi (238 psia), Fig. 5 indicates that the water may be saturated stream at 397°F. Assuming that the water *is* saturated steam, temperatures in the wellbore will remain nearly constant until all the steam is condensed as a result of heat loss. (Actually, there would be a slight change in temperature caused by a change in pressure with increased depth.) Fig. 6 also presents estimated temperatures for this case. Despite the fact that temperatures remain constant for this case, heat loss will be greater than for hot-water injection and will result in condensation of much of the steam.

To explore the possibility of reducing heat loss, assume that 500 barrels per day of hot, liquid water is injected at 1,000 psi down 2-in. line pipe centered inside the casing and that the annulus is filled with a granular insulating material. The insulating material has an effective thermal conductivity of 0.1 Btu/hr-ft-°F. The temperatures for this case (also presented on Fig. 6) show only a slight drop with depth, indicating a considerable improvement over injection down the casing.

Fig. 7 presents the percentage heat loss as a function of depth for each preceding case. Percentage loss was based upon heat content above a formation temperature of 150°F at 4,000 ft. Fig. 7 shows that 45 per cent of the heat had been lost from the injected steam by 4,000-ft depth, despite constant wellbore temperatures shown on

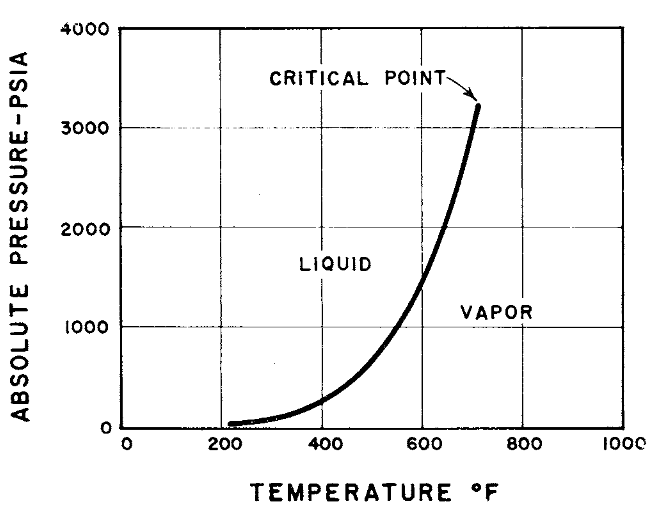

Fig. 5—Pressure-Temperature Diagram for Water.[12]

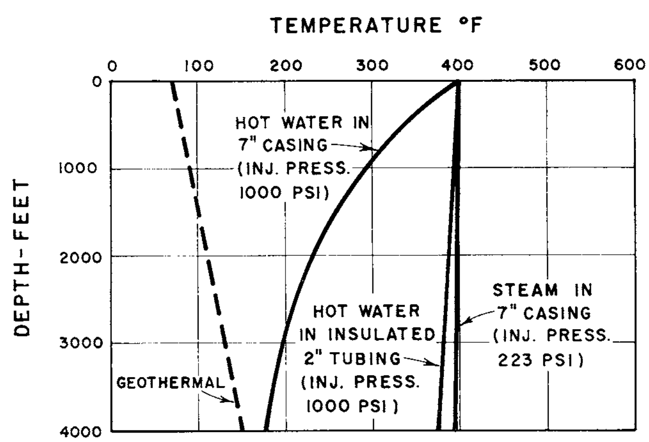

Fig. 6—Computed Wellbore Temperatures Resulting from Injection of 500 B/D of Steam or Hot Water for One Week. (Earth Thermal Conductivity of 1.4 Btu/hr-ft-°F; Earth Thermal Diffusivity of 0.04 sq ft/hr.)

Fig. 6. Heat loss during liquid water injection was reduced from 89 per cent at 4,000 ft for injection down the casing to only 9 per cent at 4,000 ft by injecting down insulated 2-in. pipe.

Fig. 8 shows the change in wellbore temperatures with increased time of injection for the previous example of injection of 500 B/D of 397°F hot water down casing. As would be expected, temperatures increase with time as the earth surrounding the wellbore becomes heated. But the thermal diffusivity of the earth is such that temperatures are still changing slowly even after 10 years of injection. This is analogous to slow pressure build-up in very tight formations.

The foregoing cases were selected to illustrate the type of information which may be gained by study of wellbore heat transmission. Because of the extreme variety of conditions possible for hot fluid injection, it does not appear feasible to compute generally applicable results. But the work required for any particular case can be done rapidly with the slide rule. If the injection project has not been drilled, it may be useful to explore the effect of tubing and casing size on heat loss. Heat loss can be reduced by slim-hole completion.

Several important observations concerning use of the methods described in this report have been made which are not apparent from the examples presented. Computed temperatures can sometimes be very sensitive to the geothermal temperatures used. Because geothermal temperatures vary considerably from field to field—and even within a given field—efforts should be made to obtain the best possible estimate of earth temperatures.

Surprisingly, good results have been obtained from different geographical areas using a single value of earth conductivity—1.4 Btu/hr-ft-°F—and a single value of thermal diffusivity—0.04 sq ft/hr. Thermal conductivity for a particular location may be estimated from field temperature logs. (See Ref. 1.)

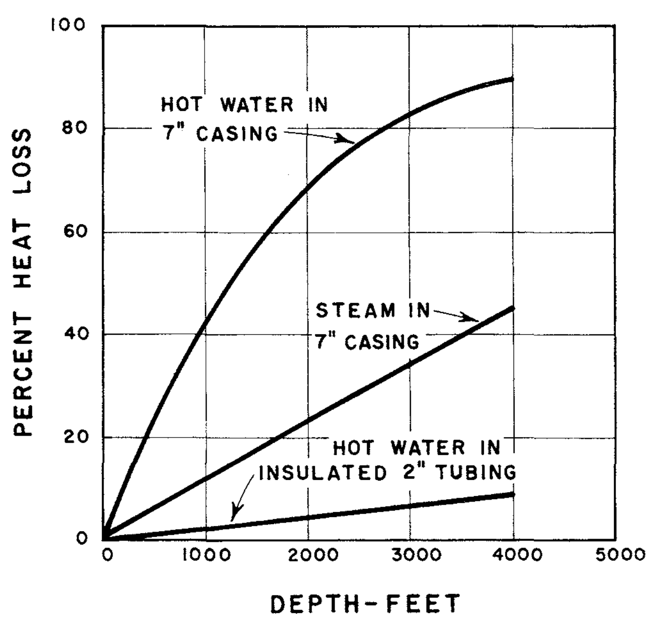

Fig. 7—Computed Heat Loss vs Depth for Injection of 500 B/D of Steam or Hot Water for One Week. (Conditions: Surface Injection Temperature, 397°F; Steam Injection Pressure, 223 psi; Hot-Water Injection Pressure, 1,000 psi; Heat Loss Based on Temperature of 150°F; Total Heat Injection Rates, 191 Million Btu/day for Steam. 44.9 Million Btu/day for Hot Water.

In most cases of water or liquid injection down casing, the resistance to heat flow between the hot stream and the earth is negligible ($U = \infty$). Thus, the bulk fluid temperature becomes equal to the casing temperature at that depth. (Both Nowak[1] and Moss and White[3] used this simplification for water-injection cases.)

Many wellbore heat problems exist which involve heat effects not considered in the subject development. Examples are: expansion of gas, heat generated by friction (an oil-well pump, for example) and latent heat effects from phase changes. Often such complications can be handled by proper modification of the solution.

In the development of Eq. 1, it was assumed that the surface temperature of the injection stream could vary with time. Because of the approximation introduced to account for heat loss to the earth, $f(t)$, surface temperature should not change rapidly. The effect of a rapid change can be pictured by considering the case of a long period of water injection at 400°F followed by a sudden drop in temperature to 200°F. It would be possible for the 200°F water to gain heat near the surface from the preheated surrounding earth, although the approximate solution would indicate a heat loss. Thus, computed results for rapid changes in injection temperature may be grossly in error and should be used with caution.

CONCLUSIONS

An approximate solution to the transient heat-conduction problem involved in movement of hot fluids through a wellbore has been developed. The approximate solution considers the effect of thermal resistances in the wellbore. These thermal resistances can be very important in certain cases. Comparison of computed temperatures with those measured in a limited number of field gas- and water-injection wells indicates good agreement. The solution may be applied to a large variety of wellbore heat problems involving different types of well completions and operating methods. Solutions to more complex wellbore heat-transmission problems may be approximated in a similar fashion with the same methods and principles. Calculations involved are simple and require only slide-rule manipulation.

NOMENCLATURE

A = a time function defined by Eq. 2, ft

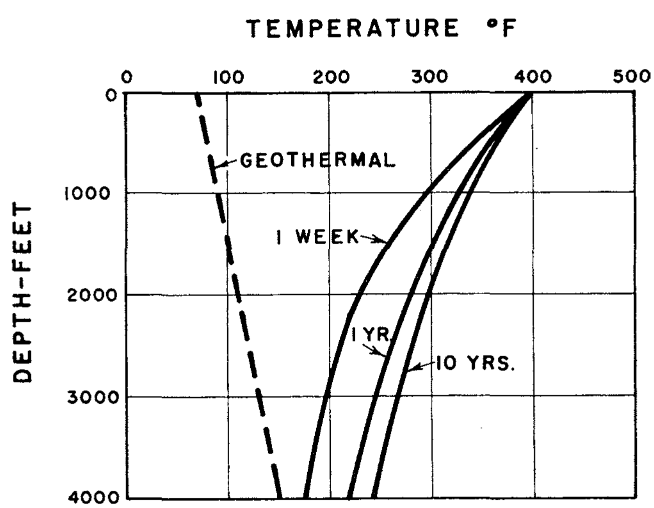

Fig. 8—Computed Wellbore Temperatures Resulting from Injection of 500 B/D of Hot Water Down 7-in. Casing.

A_1 = inside area of tubing
A_t = log mean area of tubing
A_1' = outside area of tubing
A_2 = inside area of casing
A_c = log mean area of casing
a = geothermal gradient, °F/ft
b = surface geothermal temperature, °F
c = specific heat at constant pressure of fluid, Btu/lb-°F
c_f = specific heat of earth, Btu/lb-°F
dA_a = log mean area of annulus, or log mean of A_2 and A_1'.
E = internal energy
e = base of natural logarithm
$f(t)$ = transient heat-conduction time function for earth, dimensionless (Fig. 1)
g = gravitational acceleration, 32.2 ft/sec^2
g_c = conversion factor, 32.2 ft-lb mass/sec^2-lb force
H = enthalpy, Btu/lb mass
h_1 = local film coefficient of heat transfer for gas or liquid inside tubing, Btu/day-sq ft-°F
h_2 = local film coefficient of heat transfer for gas or liquid in annulus
J = mechanical equivalent of heat, 778 ft-lb force/Btu
k = thermal conductivity of earth, Btu/day-ft-°F
k_t = thermal conductivity of tubing material, Btu/day-ft-°F
k_c = thermal conductivity of casing material, Btu/day-ft-°F
k_a = effective thermal conductivity of annulus material, Btu/day-ft-°F
0 = "on the order of"
P = absolute pressure
q = heat-transfer rate, Btu/day
Q = heat transferred from surrounding, Btu/lb-mass
r_1 = inside radius of tubing, ft
r_2 = inside radius of casing, ft
r_2' = outside radius of casing, ft
T_e = temperature of earth, °F
T_0 = surface temperature of injected fluid, °F
T_1 = temperature of fluid in tubing, °F
T_2 = temperature of outside of casing, °F
t = time from start of injection, days
U = over-all heat-transfer coefficient between inside of tubing and outside of casing based on r_1, Btu/day-sq ft-°F
U_2 = over-all heat-transfer coefficient based on outside radius of casing, Btu/day-sq ft-°F
u = fluid velocity
V = specific volume
W = fluid injection rate, lb/day
W_f = flow work, ft-lb force/lb mass
x_a = thickness of annulus or difference between inside radius of casing and outside radius of tubing, ft
x_c = thickness in casing wall, ft
x_t = thickness of tubing wall, ft
Z = depth below surface, ft
α = thermal diffusivity of earth, sq ft/day ($\alpha = k/\rho c_f$)
ρ = density of earth, lb/cu ft

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REFERENCES

1. Nowak, T. J.: "The Estimation of Water Injection Profiles from Temperature Surveys", *Trans.*, AIME (1953) **198,** 203.
2. Bird, J. M.: "Interpretation of Temperature Logs in Water and Gas Injection Wells and Gas Producing Wells", *Drill. and Prod. Prac.*, API (1954) 187.
3. Moss, J. T. and White, P. D.: "How to Calculate Temperature Profiles in a Water-Injection Well", *Oil and Gas Jour.* (March 9, 1959) **57,** No. 11, 174.
4. Lesem, L. B., Greytok, F., Marotta, F. and McKetta, J. J.: "A Method of Calculating the Distribution of Temperature in Flowing Gas Wells", *Trans.*, AIME (1957) **210,** 169.
5. McAdams, W. H.: *Heat Transmission*, Second Ed., McGraw-Hill Book Co., Inc., N. Y. (1942).
6. Jakob, M.: *Heat Transfer*, Vol. I, John Wiley & Sons, Inc., N. Y. (1950).
7. Carslaw, H. S. and Jaeger, J. C.: *Conduction of Heat in Solids*, Oxford U. Press, Amen House, London (1950).
8. Lauwerier, H. A.: "The Transport of Heat in an Oil Layer Caused by the Injection of Hot Fluid", *Appl. Sci. Res.* (1955) Sec. A, **5,** 145.
9. Marx, J. W. and Langenheim, R. N.: "Reservoir Heating by Hot Fluid Injection", *Trans.*, AIME (1959) **216,** 312.
10. "A Pilot Hot Water Flood", Newsletter, *Oil and Gas Jour.* (Feb. 8, 1960) **58,** No. 6.
11. "Steam Flooding", Newsletter, *Oil and Gas Jour.* (March 28, 1960) **58,** No. 13.
12. Keenan, J. H. and Keyes, F. G.: *Thermodynamic Properties of Steam*, John Wiley & Sons, Inc., N. Y. (1947).
13. van Everdingen, A. F. and Hurst, W.: "The Application of the Laplace Transformation to Flow Problems in Reservoirs", *Trans.*, AIME (1949) **186,** 305.
14. van Everdingen, A. F.: "The Skin Effect and Its Influence on the Productive Capacity of a Well", *Trans.*, AIME (1953) **198,** 171.

APPENDIX

DERIVATION OF WELLBORE HEAT-TRANSMISSION SOLUTION

Let us consider the injection of a fluid down the tubing in a well which is cased to the top of the injection interval. Assuming fluid is injected at known rates and surface temperatures, determine the temperature of the injected fluid as a function of depth and time. Fig. 9 presents a schematic diagram of the problem. Depths are measured from the surface. As shown on Fig. 9, W lb/day of fluid is injected in the tubing at the surface at a temperature of T_0. The inside radius of the tubing is r_1, and the temperature T_1 of the fluid in the tubing is a function of both depth Z and time t. The outside radius of the casing is r'_2, and the temperature of the casing outer surface is T_2, also a function of depth and time.

The usual procedure for flow problems of this type is to solve the total-energy and mechanical-energy equations simultaneously to yield both temperature and pressure distributions. However, the solution may be approximated by the following considerations. The total-energy equation is

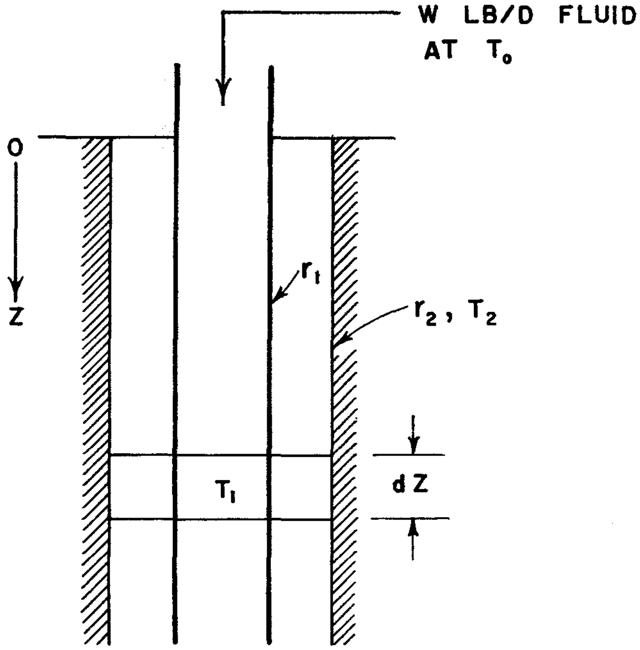

FIG. 9—SCHEMATIC OF WELLBORE HEAT PROBLEM.

$$dH + \frac{g\,dZ}{g_cJ} + \frac{udu}{g_cJ} = dQ - \frac{dW_f}{J} \quad . \quad . \quad . \quad . \quad . \quad (6)$$

Assuming steady flow of a single-phase fluid in a pipe, flow-work W_f is zero and Eq. 6 becomes

$$dH + \frac{g\,dZ}{g_cJ} + \frac{udu}{g_cJ} = dQ \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

LIQUID CASE

If the fluid flowing is a noncompressible liquid, the kinetic-energy term becomes zero. Thus,

$$dH + \frac{g\,dZ}{g_cJ} = dQ. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

But by definition, enthalpy is

$$dH = dE + \frac{d(PV)}{J} = dE + \frac{VdP}{J}. \quad . \quad . \quad (9)$$

for a noncompressible liquid. Or

$$dH = cdT + \frac{VdP}{J} \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

Neglecting the flowing friction, the VdP term is equal to the change in fluid head, and the change in enthalpy is

$$dH \approx cdT + \frac{g\,dZ}{g_cJ} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

Considering flow down the well, the increase in enthalpy due to increase in pressure is approximately equal to the loss in potential energy. Conversely, for flow up the well, the loss of enthalpy due to the decrease in pressure is approximately equal to the increase in potential energy.

As a result, the total-energy equation becomes

$$cdT \approx dQ \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

for a noncompressible liquid flowing vertically in a constant-diameter tube.

Assuming no phase changes, an approximate energy balance over the differential element of depth, dZ, yields: heat lost by liquid = heat transferred to casing, or

$$dq = -WcdT_1 = 2\pi r_1 U(T_1 - T_2)dZ. \quad . \quad . \quad . \quad (13)$$

The rate of heat conduction from the casing to the surrounding formation may be expressed as

$$dq = \frac{2\pi k(T_2 - T_e)dZ}{f(t)}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

Eq. 14 implies the assumption that heat transfers radially away from the wellbore. The time function $f(t)$ depends on the conditions specified for heat conduction and will be discussed later. Assuming the geothermal temperature is a linear function* of depth,

$$T_e = aZ + b, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

Eqs. 14 and 15 may be substituted in Eq. 13 to yield

$$\frac{\partial T_1}{\partial Z} + \frac{T_1}{A} - \frac{(aZ + b)}{A} = 0, A \neq 0 \quad . \quad . \quad . \quad . \quad (16)$$

and

$$A = \frac{Wc[k + r_1 U f(t)]}{2\pi r_1 Uk}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

An integrating factor for Eq. 16 is $e^{Z/A}$. Thus,

$$T_1 e^{Z/A} = \int \frac{(aZ + b)e^{Z/A}}{A} dZ + C(t) \quad . \quad . \quad . \quad . \quad . \quad (18)$$

or

$$T_1 e^{Z/A} = (aZ - aA + b)e^{Z/A} + C(t) \quad . \quad . \quad . \quad . \quad (19)$$

or

$$T_1(Z, t) = aZ - aA + b + C(t)e^{-Z/A}. \quad . \quad . \quad . \quad . \quad (20)$$

The function $C(t)$ may be evaluated from the condition that $T_1 = T_0(t)$ for $Z = 0$. Thus,

$$C(t) = T_0(t) + aA - b \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

And the final expression for liquid temperature as a function of depth and time is

$$T_1(Z, t) = aZ + b - aA + [T_0(t) + aA - b]e^{-Z/A} \quad . \quad . \quad . \quad (22)$$

where the time function A is defined by Eq. 17.

GAS CASE

If the fluid flowing is a perfect gas, enthalpy does not depend on pressure, and

$$dH = cdT. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

Thus, a potential-energy term will appear in the total energy balance. Eq. 13 then becomes for gas flow,

$$dq = -WcdT_1 \pm \frac{WdZ}{778} = 2\pi r_1 U(T_1 - T_2)dZ \quad . \quad . \quad . \quad (24)$$

where the plus sign on the potential-energy term is used for flow down the well and the negative sign is used for flow up the well. Simultaneous solution of Eq. 24 with Eqs. 14 and 15 yields

$$T_1(Z, t) = aZ + b - A\left(a \pm \frac{1}{778c}\right) + \left[T_0 - b + A\left(a \pm \frac{1}{778c}\right)\right]e^{-Z/A}. \quad . \quad (25)$$

The plus sign on the potential-energy term is used for flow down the well and depth taken positively increasing from the surface; the negative sign is used for flow up the well with depth taken positively increasing upward from the producing interval. Goethermal temperature must also be represented with depth increasing positively upward for flow up a well.

To apply Eqs. 22 or 25, it is necessary to evaluate the time function, $f(t)$. Eq. 14 can be rearranged to

$$f(t) = \frac{2\pi k(T_2 - T_e)}{dq/dZ}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14A)$$

which is the definition of this time function. In this form, it is clear that the function $f(t)$ has the same relationship to transient heat flow from a wellbore that the van Ever-

*It is not necessary that geothermal temperature be linear with depth. Solutions may also be obtained if geothermal temperature is represented graphically as a function of depth.

dingen-Hurst[13] constant flux $P(t)$ function has to transient fluid flow. In the case of the general wellbore heat problem, though, neither heat flux nor temperature at the wellbore remains constant except in special cases. A semi-rigorous treatment of transient heat conduction would involve a complex superposition at each depth. Thus, we wish to find approximate values of $f(t)$ which will provide engineering accuracy. Success will be determined by comparison of calculated temperatures with measured field temperatures.

Fortunately, many solutions to transient heat and fluid flow exist which may be used to estimate $f(t)$. For example, the Moss and White[3] wellbore heat-transmission solution assumes that transient heat conduction to the earth can be represented by a line source losing heat at constant flux. Carslaw and Jaeger (page 283)[7] present graphical and analytical solutions for the cases of internal cylindrical sources losing heat at constant flux, constant temperature and under the radiation boundary condition. Fig. 1 presents the time function for several different internal boundary conditions. As can be seen from Fig. 1, the solutions presented converge at long times (approximately one week or more). This is completely analogous to pressure build-up theory that at sufficiently long times pressure is controlled by formation conditions.

For times less than a dimensionless time of 1,000 (i.e., $\alpha t/r_2'^2 = 1{,}000$), the radiation boundary condition has been found to yield reasonable values for $f(t)$. The radiation inner boundary condition is

$$-k\left(\frac{\partial T}{\partial r}\right)_{r=r_2'} = U_2(T_1 - T_2) \quad . \quad . \quad . \quad . \quad . \quad . \quad (26)$$

where $U_2 = r_1 U/r_2'$. This boundary condition is analogous to the van Everdingen[14] skin effect, also well known in pressure build-up theory. Physically, Eq. 26 states that heat flow in the annular region between r_1 and r_2' is controlled by steady-state convection, rather than conduction.

The solution for this case is presented by Carslaw and Jaeger (page 282)[7] and is reproduced on Fig. 1. The time function is seen to depend upon (r_1U/k). However, the radiation boundary case does not depend strongly upon (r_1U/k) and the solution to this case approaches that of the constant-temperature cylindrical source as (r_1U/k) approaches infinity. Thus, the constant-temperature cylindrical-source solution is the recommended solution if thermal resistance in the wellbore is negligible. For times greater than those shown on Fig. 1, the line source solution as given by Eq. 5 is recommended. I am indebted to E. J. Couch for pointing out application of the radiation boundary case to evaluation of the time function.

ASSOCIATED HEAT PROBLEMS

The solution presented by Eqs. 1, 1A and 2 also applies to wellbore heat problems other than injection down tubing. For example, injection down casing may be handled by computing the over-all coefficient including only the film coefficient at the casing wall and the resistance of the casing wall. Wellbore temperatures in a flowing well may be computed if the depth scale is referenced to the producing interval. Thus, $T_0(t)$ becomes the producing formation temperature, and geothermal temperature should be expressed as a function of distance above the producing interval.

Other wellbore heat problems may be solved approximately by methods similar to those used for Eq. 1. That is, write heat balances over each flowing stream in the wellbore and assume that heat loss from the wellbore may be represented by Eq. 14. If two or more flowing streams are involved, the result will be a higher-order differential equation than Eq. 16. Temperatures in each stream may be determined, if desired. Note that Eqs. 13, 14 and 15 could have been solved for T_2, the casing temperature. This problem may have significance in interpretation of temperatures measured in the annulus when fluid is flowing in the tubing. This problem is also important when considering whether temperatures will become great enough to damage cement in hot-fluid injection wells.

SAMPLE CALCULATION FOR WATER-INJECTION WELL

For the sample calculation, the following field data will be assumed: injection rate, 4,790 BWPD; surface water temperature, 58.5°F; casing size, 7 in.-23 lb (6.366-in. ID); casing shoe, 6,605 ft; no tubing; geothermal temperature, 70.0° + 0.0083°F/ft (Z ft); and injection period, approximately 75 days.

Film heat-transfer coefficients for water flowing vertically or horizontally in tubes is correlated as a function of the Reynolds number by McAdams on page 178 of Ref. 5. The Reynolds number for flow is

$$N_{Re} = \frac{DG}{\mu}$$

$$= \left[\frac{(6.366\text{ in.})(4{,}790\text{ B/D})}{\pi(6.366\text{ in.})^2(1.1\text{ cp})}\right]\left[\frac{(350\text{ lb/bbl})(12\text{ in./ft})\,4}{(2.42\text{ lb/hr-ft-cp})(24\text{ hr/day})}\right] = 63{,}000$$

where D = inside diameter of tube, ft,
G = flowing mass flux, lb/hr-sq ft,
μ = viscosity at flowing conditions, lb/hr-ft, and
N_{Re} = Reynolds number, dimensionless;

from McAdams,[5] for Reynolds number of 63,000,

$$hD/k = 155(c\mu/k)^{0.4}$$

where k = thermal conductivity of water, 0.339 Btu/hr-ft-°F,
c = specific heat of water, Btu/lb-°F, and
$(c\mu/k)$ = Prandtl number for water, 7.5, dimensionless (McAdams, page 414 of Ref. 5).

Thus

$$h = \frac{(155)(7.5)^{0.4}(0.339\text{ Btu/hr-ft-°F})}{(6.366\text{ in.})}(12\text{ in./ft})$$

$$= 222\text{ Btu/hr-sq ft-°F}.$$

From Eq. 3,

$$1/U = 1/h + x_c/k_c,$$

since only the resistance of the water film and casing wall is involved, and the difference in inside and outside area of casing wall is neglected. The thermal conductivity of steel is about 25 Btu/hr-ft-°F. Thus,

$$\frac{1}{U} = \frac{1}{222} + \frac{(7 - 6.366\text{ in.})}{(2)(12\text{ in./ft})(25\text{ Btu/hr-ft-°F})}$$

$$= 0.00451 + 0.00106$$

$$= 0.00557,$$

$$U = 180\text{ Btu/hr-sq ft-°F}$$

$$= 4{,}320\text{ Btu/day-sq ft-°F}.$$

The transient time function $f(t)$ may be estimated from Fig. 1. The period of injection was about 75 days.

$$(\alpha t/r_2'^2) = \frac{(0.04 \text{ sq ft/hr})(75 \text{ days})(24 \text{ hr/day})}{(3.5 \text{ in./12 in./ft})^2}$$

$$= 845.$$

$$\log_{10}(\alpha t/r_2'^2) = 2.93;$$

from Fig. 1, the corresponding value of $\log_{10} f(t)$ is 0.58. Thus,

$$f(t) = 3.8.$$

From Eq. 2,

$$A = \frac{Wc\,[k + r_1 U f(t)]}{2\pi r_1 U k}$$

$$= \left[\frac{(4{,}790 \text{ B/D})(350 \text{ lb/bbl})}{2\pi(0.5)(6.366 \text{ in.})}\right]$$

$$\left[\frac{(1 \text{ Btu/lb-°F})(12 \text{ in./ft})}{(24 \text{ hr/day})(1.4 \text{ Btu/hr-ft-°F})(4{,}320 \text{ Btu/day-sq ft-°F})}\right]$$

$$\left[1.4(24) + \frac{(0.5)(6.366)}{(12)} \times (4{,}320)(3.8 \text{ Btu/day-ft-°F})\right],$$

$$A = 30{,}400 \text{ ft}.$$

Since the heat-transfer coefficient for water is large, a reasonable approximation would be that the value of U is infinite. This corresponds to the assumption that the temperatures of the water and casing are identical. The value of A computed for this case from Eq. 2A is 30,200 ft. Thus, the film coefficient for many water-injection cases should be high enough that assumption of an infinite over-all coefficient is reasonable. This will generally not be true in the case of gas injection.

Temperatures may now be computed for any depth by means of Eq. 1.

$$T_1 = aZ + b - aA + (T_0 + aA - b)\, e^{-Z/A};$$

for 6,000 ft,

$$T_1 = (.0083\text{°F/ft})(6{,}000 \text{ ft}) + 70\text{°F}$$

$$- (0.0083\text{°F/ft})(30{,}400 \text{ ft})$$

$$+ [58.5\text{°F} + (0.0083)(30{,}400)\text{°F} - 70\text{°F}]\, e^{-6{,}000 \text{ ft}/30{,}400 \text{ ft}},$$

$$T_1 = 65.2\text{°F}.$$

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