Heat transfer in Flow Through Conduits

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A common situation encountered by the chemical engineer is heat transfer to fluid flowing through a tube. This can occur in heat exchangers, boilers, condensers, evaporators, and a host of other process equipment. Therefore, it is useful to know how to estimate heat transfer coefficients in this situation.

We can classify the flow of a fluid in a straight circular tube into either laminar or turbulent flow. It is assumed from here on that we assume fully developed incompressible, Newtonian, steady flow conditions. Fully developed flow implies that the tube is long compared with the entrance length in which the velocity distribution at the inlet adjusts itself to the geometry and no longer changes with distance along the tube.

Reynolds Number

The value of the Reynolds number permits us to determine whether the flow is laminar or turbulent. We define the Reynolds number as follows.

$$\text{Reynolds number } \mathrm{Re} = \frac{DV\rho}{\mu} = \frac{DV}{\nu}$$

Here, D is the inside diameter of the tube (or pipe), V is the average velocity of the fluid, ρ is the density of the fluid and μ is its dynamic viscosity. It is common to use the kinematic viscosity $\nu = \mu/\rho$ in defining the Reynolds number. Another common form involves using the mass flow rate $\dot{m}$ instead of the average velocity. The mass flow rate is related to the volumetric flow rate Q via $\dot{m} = \rho Q$, and we can write $Q = \frac{\pi}{4}D^2 V$. Therefore, the Reynolds number also can be defined as

$$\mathrm{Re} = \frac{4Q}{\pi D \nu} = \frac{4\dot{m}}{\pi D \mu}$$

The flow in a commercial circular tube or pipe is usually laminar when the Reynolds number is below 2,300. In the range $2,300 < \mathrm{Re} < 4,000$, the status of the flow is in transition and for $\mathrm{Re} > 4,000$, flow can be regarded as turbulent. Results for heat transfer in the transition regime are difficult to predict, and it is best to avoid this regime in designing heat exchange equipment. Turbulent flow is inherently unsteady, being characterized by time-dependent fluctuations in the velocity and pressure, but we usually average over these fluctuations and define time-smoothed or time-average velocity and pressure; these time-smoothed entities can be steady or time-

1

dependent (on a time scale much larger than that of the fluctuations), and here we only focus on steady conditions when we discuss either laminar or turbulent flow.

Principal differences between heat transfer in laminar flow and that in turbulent flow

In discussing heat transfer to or from a fluid flowing through a straight circular tube, it is useful to distinguish between the axial or main flow direction, and the directions that lie in a plane perpendicular to the tube axis. In that plane, transverse heat flow can be broken into radial and azimuthal components.

The principal difference between laminar and turbulent flow, as far as heat transfer is concerned, is that an additional mechanism of heat transfer in the radial and azimuthal directions becomes available in turbulent flow. This is commonly termed "eddy transport" and is intense, providing much better transfer of energy across the flow at a given axial position than in laminar flow, wherein conduction is typically the only mechanism that operates in the transverse directions (an exception occurs when there are secondary flows in the transverse direction, such as in coiled tubes). Another difference worth noting is the extent of the "thermal entrance region" in which the transverse temperature distribution becomes "fully developed." This region is relatively short in turbulent flow (precisely because of the intense turbulent transverse transport of energy), whereas it tends to be long in laminar flow. Heat transfer correlations, based on experimental results, are typically divided into those applicable in the thermal entrance region, and those that apply in the "fully developed" region. In the case of laminar flow, it is important to be aware of this distinction, and normally a laminar flow heat exchanger is designed to be short, to take advantage of relatively high heat transfer rates that are achievable in the thermal entrance region. In the case of turbulent flow, the thermal entrance region is short, as noted earlier, and typically heat transfer occurs mostly in the "fully developed" region. Therefore, turbulent heat transfer correlations are commonly provided for the latter region.

Laminar heat transfer correlations

A variety of correlations are in use for predicting heat transfer rates in laminar flow. From dimensional analysis, the correlations are usually written in the form

$$Nu = \phi\left(\text{Re}, \text{Pr}, \cdots\right)$$

where $Nu = \dfrac{hD}{k}$ is the Nusselt number, ϕ is some function, and $\text{Pr} = \dfrac{\mu C_p}{k} = \dfrac{\nu}{\alpha}$ is the Prandtl number. Here, h is the heat transfer coefficient, k is the thermal conductivity of the fluid, and C_p is the specific heat of the fluid at constant pressure. As you can see, the Prandtl number can be written as the ratio of the kinematic viscosity ν to the thermal diffusivity of the fluid α. The ellipses in the right side of the above result stand for additional dimensionless groups such as L/D, which is the ratio of the tube length to its diameter. The heat transfer coefficient appearing in the Nusselt number is usually the average value over the heat transfer surface. You should assume this to be the case unless otherwise stated explicitly.

The Prandtl number plays an important role in heat transfer. Its physical significance can be inferred from its definition.

$$\Pr = \frac{\nu}{\alpha} = \frac{\text{Ability of a fluid to transport momentum by molecular means}}{\text{Ability of that fluid to transport energy by molecular means}}$$

Gases typically have Prandtl numbers in the range $0.7-1$, while the Prandtl number for most liquids is much larger than unity. The Prandtl number for water ranges from 4-7, while that for an oil might be of the order of 50-100. It is not uncommon to encounter Prandtl numbers for viscous liquids that are of the order of several thousand or even larger. One exception to the general rule for liquids is a liquid metal. Liquid metals conduct heat very efficiently, and therefore have a relatively large value of the thermal diffusivity α when compared with ordinary liquids. On the other hand, their viscosities are about the same as that of ordinary liquids. As a consequence, the Prandtl number of a liquid metal is typically of the order of 10^{-2}.

Now, we return to the topic of heat transfer in tube flow. As we noted before, efficient heat transfer in laminar flow occurs in the thermal entrance region. A correlation for the Nusselt number for laminar flow heat transfer was provided by Sieder and Tate.

$$Nu = 1.86\,\mathrm{Re}^{1/3}\,\Pr^{1/3}\left(\frac{D}{L}\right)^{1/3}\left(\frac{\mu_b}{\mu_w}\right)^{0.14}$$

You can see that as the length of the tube increases, the Nusselt number decreases as $L^{-1/3}$. This does not, however, imply that the Nusselt number approaches zero as the length becomes large. This is because the Sieder-Tate correlation only applies in the thermal entrance region. In long tubes, wherein most of the heat transfer occurs in the thermally fully-developed region, the Nusselt number is nearly a constant independent of any of the above parameters. When the boundary condition at the wall is that of uniform wall temperature, $Nu \to 3.66$. If instead the flux of heat at the wall is uniform, $Nu \to 4.36$, but in this case we already know the heat flux and a heat transfer coefficient is not needed. Remember that the purpose of using a heat transfer coefficient is to calculate the heat flux between the wall and the fluid. In the case of uniform wall flux, we can use an energy balance directly to infer the way in which the bulk average temperature of the fluid changes with distance along the axial direction.

Note that a ratio $\left(\mu_b / \mu_w\right)$ appears in the above laminar flow heat transfer correlation. It attempts to explicitly account for the fact that the viscosity of the fluid next to the wall is different from that in the bulk at any axial location. If the wall is hot, and we consider a liquid being heated in the pipe, the viscosity of the wall fluid will be smaller than that of the fluid in the bulk. This increases the velocity gradient at the wall and therefore the heat transfer rate compared with that obtained by assuming that the fluid is at a single temperature across the entire cross-section. Likewise, a gas that is being heated by a hot wall will be more viscous at the wall and less viscous in the bulk. The result is a decrease in the velocity gradient at the wall when compared with the case of a uniform temperature across the cross-section, and a concomitant decrease in the heat transfer rate. Similar reasoning can be used to infer the direction of change of the heat

transfer coefficient when a hot liquid or gas is cooled by a pipe wall. By explicitly introducing the ratio of the viscosity in the bulk to that at the wall in the correlation, Sieder and Tate correct the heat transfer coefficient in the right direction in all these cases.

When using the Sieder-Tate equation, all the physical properties must be evaluated at arithmetic average of the bulk average temperatures of the fluid at the inlet and the exit in calculating the Nusselt, Reynolds, and Prandtl numbers. Only μ_w, which appears in the viscosity ratio, is evaluated at the wall temperature.

The Reynolds and Prandtl numbers are raised to the same power in the laminar flow correlation. Therefore, we can write the correlation as

$$Nu = 1.86\left(\text{Re Pr}\right)^{1/3}\left(\frac{D}{L}\right)^{1/3}\left(\frac{\mu_b}{\mu_w}\right)^{0.14} = 1.86\left(\text{Pe }\right)^{1/3}\left(\frac{D}{L}\right)^{1/3}\left(\frac{\mu_b}{\mu_w}\right)^{0.14}$$

where we have introduced a new group Pe called the Peclet number.

$$\text{Pe} = \text{Re Pr} = \frac{DV}{\nu}\frac{\nu}{\alpha} = \frac{DV}{\alpha}$$

The Peclet number Pe plays a role in heat transfer that is similar to that of the Reynolds number in fluid mechanics. Recall that the physical significance of the Reynolds number is that it represents the ratio of inertial forces to viscous forces in the flow, or equivalently, the relative importance of convective transport of momentum compared with molecular transport of momentum. Thus, the Peclet number tells us the relative importance of convective transport of thermal energy when compared with molecular transport of thermal energy (conduction).

The Sieder-Tate correlation is useful only for relatively short tubes. A heat transfer correlation for laminar flow that applies regardless of the tube length, given in Mills (1) is

$$Nu = \frac{hD}{k} = 3.66 + \frac{0.065\,\text{Re Pr}\dfrac{D}{L}}{1+0.04\left(\text{Re Pr}\dfrac{D}{L}\right)^{2/3}}$$

Mills recommends that this result for the Nusselt number may be corrected for the variation of viscosity with temperature across the cross-section of the tube by multiplication by the factor $\left(\mu_b / \mu_w\right)^{0.11}$. Note that as the length becomes very large, $Nu_{average} \to 3.66$, which is the result for a uniform wall temperature when the temperature field is fully developed.

The hydrodynamic entrance length $L_{e,momentum}$ for the friction factor to decrease to within 5% of its value for fully developed laminar flow conditions can be described approximately using

$$\frac{L_{e,momentum}}{D} \approx 0.05 \, \text{Re}$$

Likewise, if the velocity profile in laminar flow is fully developed and we then apply a uniform wall temperature boundary condition, the thermal entrance length can be estimated from

$$\frac{L_{e,thermal}}{D} \approx 0.033 \, \text{Re} \, \text{Pr}$$

When both the velocity and temperature fields develop with distance simultaneously, the problem is more involved.

Turbulent flow

The entrance lengths are much shorter for turbulent flows, because of the additional transport mechanism across the cross section. Thus, typical hydrodynamic entrance lengths in turbulent flow are 10-15 tube diameters, and the thermal entrance lengths are even smaller. Therefore, for most engineering situations wherein $\frac{L}{D} \geq 60$, we use correlations for fully developed conditions. Correlations for turbulent flow are classified based on whether the interior wall of the tube is smooth or whether it is rough. Here, we discuss only the correlations for smooth-walled tubes. For information on heat transfer correlations for turbulent flow in a tube with a rough wall, you should consult a reference such as Mills (1).

Dittus-Boelter correlation

The earliest correlation for turbulent heat transfer in a smooth tube appears to have been provided by Dittus and Boelter. A common form to be used for fluids with Prandtl number in the approximate range $0.7 - 100$, and tubes with $L/D > 60$ is

$$Nu = 0.023 \, \text{Re}^{0.8} \, \text{Pr}^n$$

Here, $n = 0.4$ if the fluid is being heated, that is, if the wall is at a higher temperature than the entering fluid, and $n = 0.3$ if the fluid is being cooled. All the physical properties used in the Dittus-Boelter correlation must be evaluated at the average bulk temperature of the fluid. This is the arithmetic average of the bulk average temperatures at the entrance and the exit.

The usual recommendation is to use the Dittus-Boelter correlation for $\text{Re} > 10,000$, but in practice it is used even at lower Reynolds numbers so long as the flow is turbulent, because it is a simple correlation to use. The textbook by Welty et al. (2) provides two more correlations, one from Colburn based on an analogy between momentum and energy transport, and another from Sieder and Tate. We shall now briefly discuss the origin of the analogies.

Analogy between momentum and heat transfer

The analogies that are used for inferring results for the Nusselt number in heat transfer applications or the Sherwood number in mass transfer applications can be traced back in origin to a work by Reynolds who was examining flow and heat transfer in the context of a fluid flowing over a solid surface. Reynolds recognized that the transfer of momentum between a solid surface and a fluid flowing past it is analogous to the transfer of energy. As noted by Bennett and Myers (3), Reynolds's conclusion, when rephrased in current terminology, was that the heat transfer coefficient is proportional to the friction factor. A mathematical development eventually leads to the result

$$\frac{f}{2} = \frac{h}{\rho C_p V}$$

All subsequent analogies were extensions of this basic idea.

Colburn's j-factor analogy

The Reynolds analogy assumes, in effect, that the Prandtl number is unity, implying that the molecular transport coefficient of momentum ν is equal to the molecular transport coefficient of energy α. Several authors have developed results by relaxing this assumption. Colburn (4) defined an empirical factor

$$\boxed{j_H = \frac{f}{2} = \frac{h}{\rho C_p V} \Pr^{2/3}}$$

If the Prandtl number in the above result is set equal to unity, it reduces to the Reynolds analogy. Later in the course, we'll see that there is an analogous Colburn $j-$factor for mass transfer, j_M defined in a similar manner using the mass transfer coefficient and the Schmidt number.

Usually, in using the $j-$factor, one defines a group called the Stanton Number.

$$\boxed{St = \frac{Nu}{Re \, Pr}}$$

By substituting the standard expressions for the Nusselt, Reynolds, and Prandtl numbers in this result, we can obtain a definition for the Stanton number as follows.

$$St = \frac{Nu}{Re \, Pr} = \frac{hD/k}{(DV/\nu)(\nu/\alpha)} = \boxed{\frac{h}{\rho C_p V}}$$

Thus, we see that the Colburn analogy reduces to

$$j_H = \frac{f}{2} = \text{St Pr}^{2/3}$$

so that

$$\text{St} = \frac{f}{2}\text{Pr}^{-2/3}$$

For turbulent flow through smooth tubes, the friction factor can be correlated with the Reynolds number approximately by using an empirical result that is relatively simple to use.

$$f = 0.046\,\text{Re}^{-0.2}$$

When this result is substituted into the Colburn correlation, we obtain

$$\text{St} = 0.023\,\text{Re}^{-0.2}\,\text{Pr}^{-2/3}$$

This can be recast as a correlation for the Nusselt number to see that it is very similar to the correlation provided by Dittus and Boelter.

$$Nu = 0.023\,\text{Re}^{0.8}\,\text{Pr}^{1/3}$$

Welty et al. (2) present the Colburn equation for the Stanton number in Chapter 20 as Equation (20-29). They state certain requirements to be applied when using the Colburn equation. The Reynolds and Prandtl numbers should be evaluated using fluid properties evaluated at the average of the bulk and wall temperatures of the fluid. This means that we first take an arithmetic average of the bulk average temperatures of the fluid at the inlet and the exit, and then take an average of that value and the wall temperature. This is called the "film" temperature by the authors for historical reasons. The Stanton number, however, is to be evaluated at the arithmetic average of the bulk average temperatures of the fluid at the inlet and the exit. Welty et al. also indicate that the correlation is applicable when $L/D > 60$.

Gnielinski's correlation

It has been found that the Dittus-Boelter and Colburn equations are not as good as more modern correlations for predicting the Nusselt number in turbulent flow heat transfer through tubes. Mills (1) recommends a correlation due to Gnielinski that dates back to 1976 for smooth tubes.

$$Nu = \frac{(f/2)(\text{Re}-1000)\,\text{Pr}}{1+12.7\left(\text{Pr}^{2/3}-1\right)\sqrt{f/2}}$$

Here, f is the Fanning friction factor for a smooth tube. It is best to calculate it using the Zigrang-Sylvester equation or the Colebrook equation after setting the relative roughness equal to zero. All the physical properties appearing in the dimensionless groups in the Gnielinski correlation should be evaluated at the arithmetic average of the bulk average temperatures of the fluid at the inlet and the exit. The Gnielinski correlation also applies only to fully developed heat transfer conditions, so that we can use the same approximate criterion $L/D > 60$ that is recommended for the other correlations by Welty et al.

Sieder-Tate correlation

Welty et al. (1) also mention a correlation from Sieder and Tate for turbulent flow. You may recall the Sieder-Tate correlation for laminar flow, which we presented earlier. For turbulent flow, the Sieder-Tate equation is

$$St = 0.023 \, \text{Re}^{-0.2} \, \text{Pr}^{-2/3} \left(\frac{\mu_b}{\mu_w} \right)^{0.14}$$

You can see that this looks just like the Colburn equation, but has an additional multiplicative factor on the right side. The viscosity ratio $\left(\mu_b / \mu_w \right)^{0.14}$ also appears in the laminar flow heat transfer correlation proposed by Sieder and Tate. It attempts to explicitly account for the influence of the variation of viscosity with temperature across the cross-section on heat transfer.

When using the Sieder-Tate equation, all the physical properties must be evaluated at arithmetic average of the bulk average temperatures of the fluid at the inlet and the exit in calculating the Stanton, Reynolds, and Prandtl numbers. Only μ_w, which appears in the viscosity ratio, is evaluated at the wall temperature.

We emphasize again that the correlations given here and in the text are based on friction factor results for smooth tubes. It is possible to develop improved correlations for heat transfer in rough-walled tubes. You can consult the textbook by Mills (1) for some examples.

Non-circular cross-sections

To handle non-circular cross-sections such as annular, triangular, rectangular, and the like, we use the concept of equivalent diameter D_e. We have already provided a definition earlier in the course for this quantity, namely, $D_e = 4A/P$, where A is the cross-sectional area and P is the wetted perimeter. For a circular tube of diameter D, $A = \pi D^2 / 4$, and $P = \pi D$, so that the above definition yields $D_e = D$. For turbulent flow in non-circular cross-sections, we can use the correlations for circular tubes. The equivalent diameter is used in place of the diameter in these correlations.

References

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4. A.P. Colburn, Trans. AIChE 29, 174-210 (1933).